TIER ONE AND THE SAT

A BRIEF HISTORY

First administered in 1926 under the name Scholastic Aptitude Test, the SAT was designed to measure a person's innate mental ability as opposed to just their mastery of school subjects. The exam has undergone many changes over the years, but today it includes four sections: Reading, Writing & Language, Math with calculator, and Math without calculator. There is also an optional Essay component.

COMMON CORE & THE SAT

Funded partially by the Bill and Melinda Gates Foundation, an organization named Achievement Partners was created in 2007 by David Coleman, Jason Zimba, and Sue Pimentel. The organization was tasked with researching and developing achievement-based standards for education. Achievement Partners played a major role in developing what has come to be known as the Common Core Standards.

THE COMMON CORE

The Common Core is a set of standards that has been adopted by public schools in over 40 states around the country. Essentially, the developers of these standards strive to replace what public schools traditionally taught – the memorization of a tremendous amount of information.

The new standards are meant to ensure that schools prepare their students to be successful both in college and in their careers. They focus on critical-thinking, problem-solving, and analytical skills-building. The Common Core's goal: to teach students to think and not just to memorize.

THE NEW SAT

By 2012, the Common Core was well established. David Coleman left the organization to become the President of the College Board – the organization that writes the SAT. Within 10 months of Coleman taking over as President, the College Board announced that the SAT would change dramatically in 2016.

Based on Mr. Coleman’s background, the changes made to the SAT made a lot of sense. Taken off the test were all questions that encouraged rote memorization or lack of evidence-based answers (the essay, much of the Geometry and Vocabulary sections). Instead, the exam will test students' skills in problem-solving, analysis of texts, and their ability to find evidence for answers.

PERFECTLY CONSISTENT

The SAT is a classic “standardized test.” Standardized tests are just what they sound like: the same. Unlike tests in school where you generally know the material but are not certain in advance of the length, style, difficulty, and so on, the SAT's structure remains perfectly consistent from one date to the next.

This consistency includes not only things like the length of each section and the types of questions that show up. It goes into much more detail than that. Using the Reading section as an example:

1. The Reading section always has five passages.
2. There are always 52 questions to be completed in 65 minutes.
3. The questions do not go in order of difficulty.

WHAT'S ON THE TEST?

The SAT is made up of four mandatory sections, called tests. The sections are Reading, Writing & Language, and then two types of Math sections – one with calculator and one without. There is also an optional essay section, which many students choose to do. It lasts 50 minutes. There is no longer an "experimental" section on the exam. The below chart reflects these sections.

Section	Number of Questions	Time Allotted
Reading	52	65
Writing & Language	44	35
Math w/o Calculator	20	25
Math w/Calculator	38	55
Essay	1 Essay	50 Minutes

BREAKS

Students taking the SAT receive three breaks on test day. There is a 5-minute break after section 1, another 5-minute break after section 3, and a 2-minute break after section 4 (assuming you are taking the essay).

ORDER OF DIFFICULTY

For the Reading and the Writing & Language sections, the questions do NOT go in order of difficulty. Often the questions will appear in "chronological" order, meaning that earlier questions will align with the beginning of the passage and later questions with the end of the passage.

However, the Math sections are different. Although they are not organized perfectly in order of difficulty, they do "trend" in difficulty. That means as you progress through the section, the questions will generally get more challenging.

SCORING

As explained above, there are four core sections on the exam (ignoring the essay for now). However, when you receive your score on the SAT, you will only receive two scores. The first is for Math and the second is called Evidenced Based Reading and Writing.

The way the Math score is calculated is pretty straightforward. You count the total number of questions you get correct from both Math sections and then compare it to scores on that section's unique "Raw Score Conversion Table," an example of which is shown below. This will give you the familiar score of 200-800 points.

For the Evidenced Based Reading and Writing, first you count the number of questions correct from the Reading section, and then you look at the Raw Score Conversion Table to convert your Reading raw score to a score of 10-40. Next, you do the same for the Writing and Language section, getting another score of 10-40. Finally, you add these numbers together for a score of 20-80 and add a zero for the familiar 200-800.

Using this methodology, you end up with a score from 200-800 points each in Math and in Evidenced Based Reading and Writing for a total composite score of 400-1600 points. See below for an example.

RAW SCORE CONVERSION TABLE SECTION AND TEST SCORES								
Raw Score	**Math**	**Reading**	**Writing & Language**		**Raw Score**	**Math**	**Reading**	**Writing & Language**
0	200	10	10		30	530	28	29
1	200	10	10		31	540	28	30
2	210	10	10		32	550	29	30
3	230	11	10		33	560	29	31
4	240	12	11		34	560	30	32
5	260	13	12		35	570	30	32
6	280	14	13		36	580	31	33
7	290	15	13		37	590	31	34
8	310	15	14		38	600	32	34
9	320	16	15		39	600	32	35
10	330	17	16		40	610	33	36
11	340	17	16		41	620	33	37
12	360	18	17		42	630	34	38
13	370	19	18		43	640	35	39
14	380	19	19		44	650	35	40
15	390	20	19		45	660	36	
16	410	20	20		46	670	37	
17	420	21	21		47	670	37	
18	430	21	21		48	680	38	
19	440	22	22		49	690	38	
20	450	22	23		50	700	39	
21	460	23	23		51	710	40	
22	470	23	24		52	730	40	
23	480	24	25		53	740		
24	480	24	25		54	750		
25	490	25	26		55	760		
26	500	25	26		56	780		
27	510	26	27		57	790		
28	520	26	28		58	800		
29	520	27	28					

GUESSING vs SKIPPING ON THE SAT

As described above, the way the exam is graded is simply by counting the number of correct questions and comparing it to the curve table. That means that there is NO PENALTY for getting wrong answers on the SAT. For this reason, you need to answer EVERY single question on the test even if that means you close your eyes and point!

WHAT THE SCORES MEAN

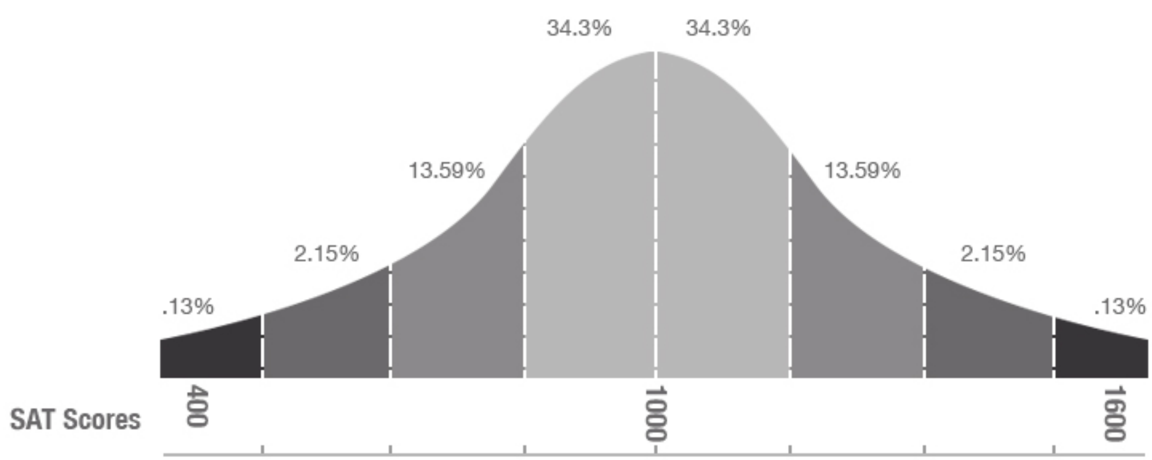
The SAT is scored on a curve, as we will explain in more depth below. Practically speaking, it means that your score is not calculated like at school where you simply compute the percentage of questions you get correct. Instead, as the chart above showed, each test has a unique scoring scale.

As you can see in the chart below, each scaled score you get can be converted in to a "percentile." That number represents the percentage of students that you scored above. For instance, if you scored in the 90th percentile, it means you did better than 90% of test takers.

UNDERSTANDING THE CURVE

The SAT, like the ACT, is graded on a forced curve. That means the grading of the test is not simply based on your personal performance. Rather, it is based on how you do relative to your peers.

SAT SCORE DISTRIBUTION



The curve the SAT uses changes for each exam date. Therefore, if you take the March administration of the test, you will be compared only with the other students around the globe that took the exam that day.

Practically speaking, the curve also means you can get the same number of questions correct on two exams but end up with significantly different scores.

A REAL LIFE EXAMPLE

Below are examples from two real tests. You can see that in Example 1, if you were to get 42 Math questions correct, you would get a score of 630. Yet on Example 2, those same 42 questions correct would yield a score of a 660.

Example 1

Math	Scale Score
58	800
57	790
56	780
55	760
54	750
53	740
52	730
51	710
50	700
49	690
48	680
47	670
46	670
45	660
44	650
43	640
42	**630**
41	620
40	610
39	600
38	600
37	590
36	580
35	570
34	560
33	560
32	550
31	540
30	530
29	520
28	520
27	510
26	500
25	490
24	480
23	480

Example 2

Math	Scale Score
58	800
57	790
56	780
55	770
54	760
53	750
52	740
51	730
50	720
49	710
48	700
47	690
46	690
45	680
44	670
43	660
42	**660**
41	650
40	640
39	630
38	620
37	610
36	600
35	590
34	580
33	570
32	570
31	560
30	560
29	540
28	530
27	520
26	510
25	510
24	500
23	490

PRACTICAL CONSIDERATIONS OF THE CURVE

The unique nature of the bell-shaped curve is that if you are scoring anywhere near the center average, it takes more correct responses to questions to raise your score and fewer wrong responses to lower your score. That's because in a bell-shaped curve, test graders are trying to keep as many students as possible near the center of the curve. They accomplish this by making it take more questions right and wrong to change your score.

Math	Scale Score
58	800
57	790
56	780
55	760
54	750
53	740
52	730
51	710
50	700
49	690
48	680
47	670
46	670
45	660
44	650
43	640
42	630
41	620
40	610
39	600
38	600
37	590
36	580
35	570
34	560
33	560
32	550
31	540
30	530
29	520
28	520
27	510
26	500
25	490
24	480
23	480

If you take a look at the example chart here (once again from a real SAT), you will see a perfect example of this. In the scores ranging just 120 points from 480-600, there are four times that getting an extra question correct does not move your score at all. In other words, if you are currently getting 23 questions right, and you learn an extra concept and then get 24 questions correct, the score still stayed at a 480. The same happened from 28 to 29, 33 to 34, and 38 to 39.

Juxtapose that pattern to scores of 610-700. Here you only have this phenomenon occur once from 46 correct answers to 47. Besides that, every question is worth 10 points. That makes it easier to raise your score.

However, once you score a 710 or above, notice how all of a sudden we change patterns once again. Now there are two separate times where you can actually jump 20 full points with just one right answer! It happens between a 51 and 52, as well as between 56 and 57. This makes it even easier to get a large score improvement.

To put this in perspective, if a student is getting 23 questions correct, he scores a 480. If he studies exceedingly hard and learns how to get 16 more questions correct he will score at a 600 – a 120 point improvement. However, if he is getting 42 questions correct to begin with for a score of 630, and he studies really hard and gets 16 more questions correct, he will get a score of 800 – a 180 point improvement. Therefore, there are 60 more points gained with the equal number of questions scored correctly.

EXPECTATIONS AND GOALS

It is not possible to establish a "normal" improvement goal on the SAT. The truth is that setting a goal depends on many factors: how much work a student does, how much assistance they are getting, what score they start off at, why they got their original score, etc.

That being said, we do encourage two simple approaches when creating your goals.

First, re-imagine your current test score as being out of your goal score instead of out of 1600. For instance, if you are currently getting a composite score of 1100 and would really like to get a 1300, tell yourself, "I'm getting a 1100 out of 1300 and only need 10 questions right per section for a 'perfect' score." If you continue to think of your score as being out of a total of 1600, the gap between your achievement and that "perfect score" can be demoralizing. A perfect score requires every answer to be right. That is not only an unrealistic goal for most, but not necessary for acceptance at the vast majority of colleges.

Second, you should have a two-tiered goal. In other words, make an initial modest goal of just a 100- or 150-point improvement, and then you can have a second, more aggressive, goal. So again, using the above example, if you earned a 1100 to begin with, you may have an initial goal of 1200 and a secondary goal of 1300.

The reason this second approach is critical is because you will study differently to just earn 100 points versus earning 200. Maybe you would be more comfortable guessing on a few extra questions per section or something similar. We want to see students striving for modest improvements along the way instead of constantly trying to "hit a homerun."

HOMEWORK – NOT JUST AN ASSIGNMENT

We view homework as one thing, and one thing only: an opportunity to help a student achieve independent mastery of material. That's it. It is NOT about completing a certain number of questions or sections. Instead, we want you to start each week with a specific goal in mind. Once you have your goal established, you will work until you hit it. If that takes 20 minutes, then you got lucky! But if that takes 3 hours, then that is what needs to get done.

PRACTICE TESTS

As discussed above, scores have a tendency to go up and down depending on the curve of each test. Therefore, the concern is that a student who is working really hard and accomplishing a lot may get dismayed by a "falling" score and then change up their prep regimen.

Nonetheless, practice tests still are very important learning tools. ***<u>Generally</u>*** speaking, the below chart is what we recommend for students in terms of how many tests they should take based on how much preparation they are doing.

Initial Hours of Instruction	Hours Left Before 1st Test Completed	Hours Left Before 2nd Test Completed	Hours Left Before 3rd Test Completed	Hours Left Before 4th Test Completed	Hours Left Before 5th Test Completed
8 Hour	6	3	N/A	N/A	N/A
18 Hour	14	9	4	N/A	N/A
24 Hour	20	15	10	5	N/A
30 Hour	24	18	12	6	3

RULES FOR TAKING TESTS

Here are the three Tier One Rules for Practice Tests:

Rule 1: All practice tests must be taking in ONE sitting. You can drastically impact your score by taking one half of a test on a Saturday and the second on a Sunday, or even by just taking a two-hour break because you got tired. You are allowed only one 10-minute break between sections 2 and 3.

Rule 2: All sections of each test must be timed accurately. At the beginning of each section, it is clearly printed in the test booklet how much time you are allotted for each section. Make sure not to move on early or late.

Rule 3: You must try hard on all tests. We make critical decisions about preparation, homework, goals, etc. based on these tests. Therefore, it is important that you take them seriously. This primarily means NO cell phones or text messaging during the test. This also means no taking phone calls, watching the football game, eating lunch, etc. You want to pretend that each practice test is the real test.

EXTENDED TIMING

For students with various learning challenges and exceptional situations that could affect their test-taking ability, the SAT allows alternate testing accommodations. Sometimes this will come in the form of different testing conditions, such as the ability to fill out answers in the test booklet instead of bubbling-in on the Scantron form. Often, it will come by way of extended time for the exam.

The most common form of accommodation is additional time for testing – generally "time and a half." In this situation, a student is simply allotted 50% more time than normal for each section. That means 97.5 minutes for Reading, 52.5 minutes for Writing & Language, 37.5 minutes for Math without calculator and 82.5 minutes for math with calculator.

There are some situations where that is different. One example is that the College Board sometimes only approves students for extended time in one section – like Math only. The second example would be when a student is given "double time" for the sections. When that happens, the test is actually taken over two days instead of one.

CUMULATIVE REVIEW

One of the issues with test prep is that it tends to be done over a long period of time. This leads to the obvious concern that skills learned early on in the process will become rusty or that students will forget a lot of the information they learned earlier in its entirety.

In order to avoid this common problem, we implement a simple system of cumulative review. The idea is that after each session, you should not only do that week's homework, but also practice at least SOMETHING from every topic studied so far.

Sample Review Schedule

Week	Material Covered	Review
1	1st half of grammar	1st half of grammar
2	2nd half of grammar	harder parts of 1st half of grammar and all of 2nd half
3	Math Tricks	harder parts of grammar and math tricks
4	1st half of math review	easier parts of grammar, math tricks and 1st half of math
5	2nd half of math review	some grammar, 1st half of math tricks and parts of math review
6	Reading method	some grammar, 2nd half of math tricks, some math and reading method

TIER ONE “CORE SKILLS”

We will be discussing unique methods and reviews for each of the sections on the exam. However, we must first start with the “Tier One Core Skills.” These are three unique skill set methods that Tier One developed which are applicable to all sections of the exam.

CORE SKILL 1: PACING TIMING

The SAT is famous for being very difficult to deal with in terms of timing. Further, the SAT covers quite a bit of information. This makes timing even more of an issue.

PACING VS TIMING

There are *two* issues when it comes to timing: there is the ability to finish a section in time, and there is the ability to be in control throughout a section and not have to rush near the end to finish.

Finishing a section on time AND in control is an obvious benefit. As stated earlier, it is imperative that you guess on EVERY single question even if that means you have to close your eyes and point! That being said, we much prefer that you carefully consider each question and choose the best answer as opposed to randomly guessing.

PRACTICE *DOESN'T* MAKE PERFECT

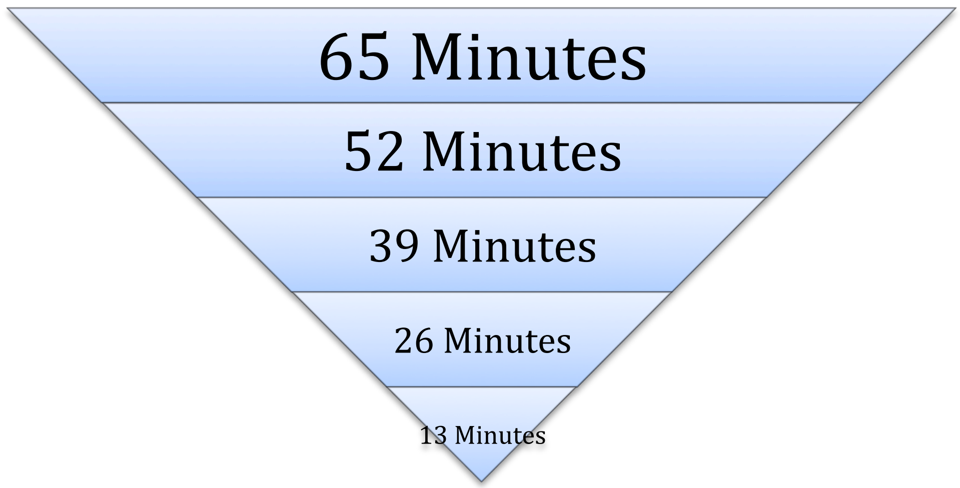
There is a common expression that “practice makes perfect.” Here at Tier One we agree that practice certainly makes “better,” but it hardly makes “perfect.” That is why we encourage our students to “practice perfectly.” Practically speaking, this means doing all your practice in a way that is intelligent and organized in order to hit a specific goal.

THE TIER ONE TIMING METHOD

How will we cure your timing and pacing problems? *Perfect practice*.

Perfect practice means having an organized and steady approach to your timed practice. The Tier One Timing Method will provide the outline needed to accomplish this goal. We implement it on each section a bit differently, but let's see an example from the English section to illustrate.

Reading Section with One Passage Gradation



In the above graphic, each of the levels represents a passage in the Reading section. It starts off at the bottom with just one passage, the next level is two passages, and on up until at the top it represents all five passages from an English section.

Notice that you will start off small with just one passage and you will allot 13 minutes (the average amount of time you have per passage) for your practice. You are going to do this over and over again until you can successfully complete the passage in 13 minutes. Successful completion is defined by not feeling pressed for time while also providing accurate answer choices.

Once you can do one passage in 13 minutes, you will graduate to two passages in 26 minutes. You will continue to do this until you reach mastery, and then you will move on to three passages in 39 minutes, and so on, until you can do a full section in 65 minutes.

When done correctly, this method allows you to slowly build up to total mastery over a section's timing. Although you will start off completing just portions of a section while timed, once you have achieved mastery over that amount, you will keep adding on more and more until you can do full sections this way.

CORE SKILL 2: PERFECT PRACTICE

The second core skill of "perfect practice" is quite dynamic. We'll break it into two critical components: The first is the Tier One Three Point Annotation System, and the second is the Tier One Codification Method. The underlying value of this core skill is organized and controlled study that allows students to start to study smart.

TIER ONE THREE-POINT ANNOTATION SYSTEM

Although the SAT is a standardized test, you will NEVER see any particular question more than once. Therefore, when reviewing completed assignments, don't simply ask, "How can I get THIS question correct?" Instead, ask, "What lesson can I learn from this question that will help me to get a similar one correct in the future?"

When reviewing completed assignments and practice tests, most students and tutors review the material – that is smart. However, they choose to review only the questions the student got wrong – that is NOT smart. There are a few reasons for this. First, just because you got a question correct does not mean that you got it right for the right reason. This is a multiple-choice test, so it's very possible that you simply guessed correctly.

Another reason why we are cautious about this style of review is because it assumes that every question you got wrong, you must have approached wrong. That is also not true. Sometimes your approach was correct, but you simply made a small mistake of some kind – for example, you added incorrectly or misunderstood wording. That is relatively hard to control.

THE SYSTEM

Whenever doing ANY practice problems – that means full-length practice tests, practice sections, and groups of practice questions, as well as all questions you do in-session with Tier One – annotate each question individually. Your goal is to make a small note on each question directly after you complete it. The note will be a 1, 2, or a 3.

Note	Meaning
1	Means "I completely understood this question and am comfortable with the answer I chose."
2	Means "I neither had mastery nor was I totally lost – rather I was something in between."
3	Means "I am very uncomfortable and do not feel confident with the answer I chose."

Instead of simply reviewing only questions you got wrong, we should go over everything that you marked a 2 or a 3. The idea here is that if you were not confident with this question, there is a good chance you will not be confident with a similar question on the real exam. That means we need to go over all such questions REGARDLESS of whether or not they were correct or incorrect.

TIER ONE CODIFICATION METHOD

The second part of our method is meant to help us optimize how much we can learn when reviewing material. During this simple "codification process," we take notes on the information you learn from the "2s" and "3s" you review.

Your review sheet contains the notes of why each question you reviewed was not considered a "1" when you were answering it. (For instance, it could be you made a careless error, it could be you got tricked by a classic wrong answer choice, or maybe you didn't know a certain grammar rule.)

Example for Writing and Language Assignments

Assignment 1	**Assignment 2**	**Assignment 3**	**Assignment 4**
-*Subject/Verb Agree.*	-Misread question	-Semicolons	-Danglers
-Misread question	-Verb Tense	-Antecedent	-Parallelism
-Misread question	-Misread question	-Comma	-Comma
-Subject/Verb Agree.	-Comparatives	-Misread question	-Author's Goal
-Antecedent Agreement	-Comma	-Good vs well	-Passive Voice
-Semi-colon	-*Subject/Verb Agree.*	-Danglers	
-Comma	-Comma	-*Subject/Verb Agree.*	
-Comma	-Colons	-Ran out of time	
-Ran out of time	-Ran out of time		

STUDYING WITH YOUR SHEET

Now that you have created this sheet, there are two more simple steps to take.

Step 1: Actively review the sheet before starting your next assignment. This will allow the information you struggled with last time to be in the forefront of your mind on the next assignment.

Step 2: Compare the notes from your current assignment to the notes you made on previous assignments and look for patterns. Whenever you see something that shows up two times, make a note to yourself to proactively look for it in future assignments. If you see that same issue three times or more, you should take time to focus on mastering that one particular grammar rule, math concept, method, or issue.

From the above example, you can see a few different patterns. First, notice how we put in italics "Subject/Verb Agreement." This was the first mistake made on Assignment 1, and it was also made in Assignments 2 and 3. This student probably noticed the issue after Assignment 2, since it had popped up both times, and then certainly after Assignment 3. That is also why it didn't come up again on Assignment 4. Our student put in a lot of hard work after the third assignment to really get rid of the issue.

Clearly, there are other patterns here, too, like running out of time, commas, etc. The idea here is that after each assignment, you find the overlap, work on mastering it, and do not make the same mistake in the future.

CORE SKILL 3: ANALYTICAL GUESSING

The SAT does not penalize students for getting a wrong answer. That means that students must answer every single question on the exam. However, just because you should answer each and every question doesn't mean you should randomly guess.

RANDOM GUESSING & PROBABILITY

Most questions on the SAT have just four answer choices to choose from. That means that if you get stuck on a question, you could literally close your eyes, point to a random answer choice, and you would have a 1 in 4 shot of guessing correctly!

Although random guessing is better than skipping questions, it is hardly your best tool for the exam. Instead, we teach a system called "Analytical Guessing." We want to be smart about how we go about guessing so we can increase our odds of getting a question correct.

For instance, imagine if you knew definitively that answer choice "A" was incorrect. In that case, if you were to randomly guess between B, C, and D, you would actually have a 1 *in 3* shot of guessing correctly.

"CONSERVATIVELY" vs "LIBERALLY"

The Tier One system of Analytical Guessing requires students to approach answer choices twice. The first time we go through choices "conservatively," and the second time we go through "liberally."

When you are unsure of an answer to a question, the first thing you will do is go through your choices conservatively – meaning you only eliminate questions that you are 100% sure are incorrect. Any choices that you are not positive about or about which you have some misgivings need to stay.

The next step is to go through your remaining choices, this time being more liberal about which choices you eliminate. At this point, you are comparing answer choices to each other. It's not necessarily about which one choice is right; it's about which choice is best *relative to the others*.

IN SUMMARY

The sum total of our method is simple. We have now established a system of reviewing questions that allows you to learn with each assignment performed. You will benefit from not simply going over questions you got wrong, but instead finding problem areas even in questions you got right. Further, you will have simple notes on all of your weaknesses so you can review them directly before doing your next assignment. Finally, you will be able to recognize patterns of problem areas so you can study with focus.

At the end of the day, all of the Tier One Methods are meant to help you study smart and study efficiently. Now that we have gone through all the background you need about the test, as well as the Tier One Core Skills, let's dive in to the specific methods and reviews that you will need for each of the sections on the exam.

READING SECTION

Remember that your Evidenced Based Reading and Writing score (200-800 points) is made up of two sections – Reading and Writing & Language. The section is scored by giving 50% of the weight to Reading and 50% to Writing & Language. This grading scale has practical consequences.

The Reading section is made up of 52 questions while the Writing and Language has just 44 questions. Yet, the two sections have equal weight for the scoring. That means 44 questions are somehow equal to 52!

Practically speaking, that means that each Writing & Language question has more influence on your overall score than Reading questions. As a real life example, in order to get a 35/40 score on each section, you would need to answer FIVE more questions correct on the Reading section than you would on the Writing & Language section.

WHAT'S BEING TESTED

The Reading section has a few qualities that are ALWAYS true:

1) It is always the first section on the test.
2) There are always five passages.
3) You will find the following passages:
    a. One passage on a classic or contemporary work of literature
    b. One passage or pair of passages from either a US founding document or a text in the great global conversation
    c. One passage about economics, psychology, sociology, or another social science
    d. Two science passages (or one pair of passages) about earth science, biology, chemistry, or physics.
4) There are always 52 questions.
5) The questions do NOT go in order of difficulty.
6) There is NO penalty for wrong answers, so make sure you answer every single question even if you have to guess.

The Reading section is quite different than the Math. For this section, there is NO prior knowledge needed in order for you to get every question correct – even for the science passages. In other words, all the information you need to know to "ace" the section is right there in the passage. Therefore, test-taking skills and some simple methods are what you need for mastery.

THE TIER ONE PACING METHOD

As alluded to earlier, we want to make sure that all practice is done timed. As a reminder, here are the rules for you should progress through your timed practice over time.

Reading Section with One Passage Gradation

In the above graphic, each of the levels represents a passage in the Reading section. It starts off at the bottom with just one passage, the next level is two passages, and on up until at the top it represents all four passages from a Reading section.

Notice that you will start off small with just one passage and you will allot 13 minutes (the average amount of time you have per passage) for your practice. You are going to do this over and over again until you can successfully complete the passage in 13 minutes. Successful completion is defined by not being pressed for time while also providing accurate answer choices.

Once you can do one passage in 13 minutes, you will graduate to two passages in 26 minutes. You will continue to do this until you reach mastery, and then you will move on to three passages in 39 minutes, and so on, until you can do a full section in 65 minutes.

Starting from the beginning of your prep, you will want to make sure you do timed practice in this sequence. The goal is not just that you learn how to finish a section on time, but also to gain mastery over your timing so you don't feel pressed for time while completing a section either. This should stop you from making critical careless errors and from experiencing heavy testing anxiety, as well.

TIER ONE METHOD FOR READING

Step 1:	Read the passage "actively."
Step 2:	In one sentence, write a synopsis of the passage.
Step 3:	Evaluate question type.
Step 4:	Methodically eliminate answer choices.
Step 5:	Check your answer against the question.

STEP 1: READ THE PASSAGE "ACTIVELY"

There are essentially two ways to read a passage: You can read it "actively" or "passively." Passive reading is when you read without a purpose. Most students would think of this in the context of reading a text message or some other non-important piece of writing. You are reading, but not with a plan in mind.

Unfortunately, the truth is that this is how most students read academic works, as well. Reading passively can be applied to a situation where you are reading and can afford to allow the passage to dictate what you pay attention to. For instance, if you were reading a Biology textbook, you may read line-by-line and only stop if something happens to strike you as important. You weren't searching for that information – it just jumped out at you.

Active reading, on the other hand, is reading for a purpose. A simple example of this would be if you were looking through an article specifically trying to find someone's name. You would "active" search for something when reading as opposed to passively reading.

The takeaway here is that when reading for the ACT, you want to stay present while reading. Often the passages are full of details, characters, facts, settings, etc., and it's really easy to get lost in it all. That is the issue with reading passively. If you just let whatever strikes you as important take your attention away, you'll have a lot of timing problems.

STEP 2: IN ONE SENTENCE, WRITE A SYNOPSIS OF THE PASSAGE

So what exactly are we "actively reading" for? In other words, what are you searching for when reading? Essentially, you want to make sure that you paid attention to the general structure of the piece. You want to be able to succinctly write a synopsis of the passage in just ONE sentence.

As long as you read actively in Step 1, writing this synopsis should be simple. Don't make yourself crazy by trying to write the perfect sentence or by writing a synopsis paragraph. Just keep it simple. The idea here is that as long as you have a handle on the gist of the passage, you will be able to answer questions more freely, find relevant parts of the passage quicker, and feel more confident as you go.

STEP 3: EVALUATE QUESTION TYPE

Below are the five most common and easiest to prepare for question types on the Reading section.

MAIN IDEA QUESTIONS

These will be characterized by phrases such as: "the author's main point," "which would be the best title," or "the purpose of the passage is to."

Answer choices to avoid are:

1) Ones that refer to specific details about the passage
2) Answer choices that reference facts that are in only one portion of the passage
3) Answer choices that are contrary to the author's opinion.

Conversely, answer choices that are *potentially* correct might be:

1) If you see a choice that seems to combine ideas from all over the passage
2) Choices that are in line with the author's opinion
3) Answers that seem to be worded in a relatively general fashion.

INFERENCE QUESTIONS

The types of words you would look for in these questions would be words like "implied," "suggests," "inferred," "most likely," and "primarily."

Answer choices to avoid are:

1) Answer choices that seem to piece together a lot of different ideas
2) Answer choices that reflect general concepts not specific ones.

Correct answer choices to look for will be:

1) Geared towards a specific piece of information
2) Taken from one particular section or paragraph of the passage
3) Something matching a note you had written by that part of the passage.

DETAIL QUESTIONS

Example prompts would be: "in order to" (which is the most common), or "because of what factor?"

Answers to watch out for on Detail questions would be ones that:

1) Don't align very tightly with a quote from your passage
2) Seem to stray from the passage or make inferences.

Answers that are good will be characterized by closely lining up with specific information or quotes from the passage.

VOCABULARY IN CONTEXT QUESTIONS

Vocabulary in Context questions always say something like: “The word X most nearly means what?”

Answers choices to watch out for will be ones that are primary definitions of words – usually the correct answer will NOT be the most common definition of the word in question.

EVIDENCE/PAIRED QUESTIONS

Paired questions start off with a traditional question (usually an Inference or a Detail question). The next question will say: “Which choice provides the best evidence for the answer to the previous question?”

The first thing you will want to do when coming across the evidence question is immediately go back to the previous question and double check for accuracy. Unlike normal questions where if you made a mistake it only effects that one question, here a mistake will effect both the initial question and the Evidence question that follows.

Answers to watch out for are ones that are evidence of something important from the passage, just not SPECIFICALLY for the previous question.

STEP 4: METHODICALLY ELIMINATE ANSWER CHOICES

We want you to have a goal in mind when you go through each of your four choices. First check each answer choice “conservatively” then “liberally.” “Conservatively” means that you are evaluating answers based on absolute confidence about the choice. Only get rid of answer choices if you are 100% certain that they are wrong. Once you have gone through them conservatively, then you can circle back around to go through them “liberally.”

Going through answer choices liberally means that you are going to compare each answer choice you have left to each other in order to figure out which one is the “best” answer.

TRUE…BUT FALSE

Watch out for answer choices that are absolutely true, but don’t really relate to the question being asked.

EXTREME

Another thing to look out for: answer choices that are too extreme. Usually answer choices that are too specific, too absolute, or extreme are incorrect. These are not always incorrect, but you should be extra careful when evaluating them.

TO INFER OR TO ASSUME, THAT IS THE QUESTION!

An inference, as defined for the SAT specifically, is when you can say that because of a statement X in a passage, you can infer that Y ***must*** also be true. An assumption is when you say because of X, it’s ***possible*** that Y is true.

To give an analogy, if we were to say, “The grass is wet,” you can assume the sprinklers were on or that it rained because it’s ***possible*** that is true. We can infer that because the grass is wet, it is therefore not dry. **You can and should infer, but you should NOT assume.**

PAIRED PASSAGES

Generally you will find one set of "paired" passages on the SAT. Paired passages are simply two unique passages written by different authors, but which consider a similar subject. You will then be asked some questions just about Passage One, some questions just about Passage Two, and then some questions about both.

TIER ONE METHOD FOR PAIRED PASSAGES

Step 1:	Actively read Passage One.
Step 2:	Methodically answer questions pertaining to Passage One.
Step 3:	Actively read Passage Two.
Step 4:	Methodically answer questions pertaining to Passage Two
Step 5:	Methodically answer the questions that compare and contrast the two passages.

STEP 1: ACTIVELY READ PASSAGE ONE

Just like with a single passage, you should actively read the first passage (exactly as described above) and write your one-sentence synopsis.

STEP 2: METHODICALLY ANSWER QUESTIONS FOR PASSAGE ONE

This step is a little more all-encompassing than previous ones. However, it is basically a combination of Steps 3, 4 and 5 from the Tier One Method for Reading: evaluate question type, methodically answer, and check your work.

The important thing to do here is to sift through the questions and *only* deal with those that relate to Passage One. It is easy to find these, as they will either say "Passage One" or they will give a line reference number found in that passage.

STEP 3: ACTIVELY READ PASSAGE TWO

There are a few additional things you want to keep in mind while reading this second passage. First off, try to think about how the author of the second passage seems to agree or disagree with the opinions of the first author. Also, in your synopsis, you should note any ways that the two passages obviously relate to one another.

STEP 4: METHODICALLY ANSWER QUESTIONS FOR PASSAGE TWO

Once again, this step is the same as Step 2. The only difference is that you must look for the questions that mention Passage Two or use line reference numbers from there.

STEP 5: METHODICALLY ANSWER COMPARE/CONTRAST QUESTIONS

This step is really the only truly unique step in the paired passage method. There are always a few questions in the paired passage section where you will have to compare and contrast the two passages you read. The good news is that there are generally only two types of these questions to worry about.

COMPARE

These will ask you to evaluate how the two passages have something in common or simply how the two relate to each other. Sometimes, you will need to consider if the information in the answer choice holds true for ***both*** passages. Other times, you will need to make sure that the comparison being drawn between the two is actually true.

For instance, you may be asked about the relationship between the two passages and an answer choice will say "Passage Two refutes the central claim advanced in Passage One." This is where the synopsis you wrote for each passage comes in handy.

In each of these instances, you are just comparing the two passages.

CONTRAST

The second type of question is a little harder to understand. This question will be worded similarly to the following: "What would the author of Passage Two think about the statement made on lines 2-4 of Passage One?" or "What would the author of Passage One think about the statement made on lines 45-48 of Passage Two?"

The important point here is that you need to first make sure you have clarity on the quoted piece and only then evaluate that point from the perspective of the author of the passage not being quoted. For instance, if it asks what the author of Passage Two thinks about a statement made in Passage One, you would first clarify the meaning of the statement in Passage One and then your answer choice will evaluate that topic from the point of view of the author of Passage Two.

SCIENCE PASSAGES

The last issue on the Reading section has to do with science passages. As stated earlier, you will see two of these types of passages – or one paired passage – that deal with earth science, biology, chemistry, or physics.

Essentially, these passages test your ability to read and interpret charts, graphs, and tables in relation to a reading passage about a topic in science. For this section, there is effectively no prior knowledge needed in order for you to get every question correct. Around 95% of the information you need to know for the section is in the passage or chart.

For these passages, you will follow the same reading methods outlined earlier in this chapter. The only distinction is that you will need to be more familiar with reading and interpreting graphs and tables. Below is some instruction on how to sharpen those skills.

HOW TO READ GRAPHS AND TABLES

GRAPHS

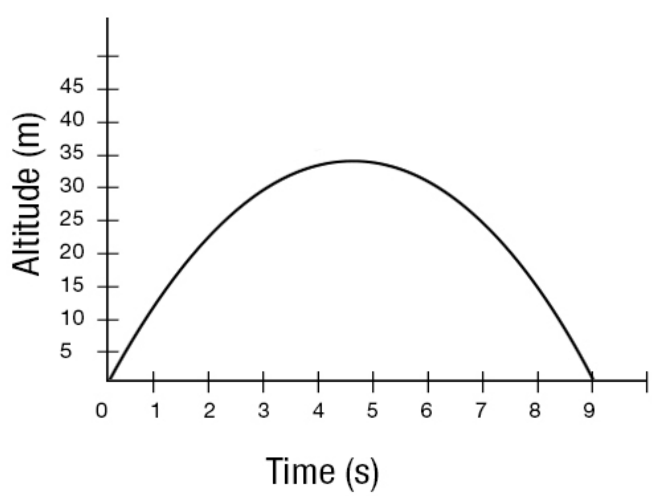
Graphs tend to have an x (horizontal) and y (vertical) axis.

The horizontal axis (x) tends to (but does not always) correspond with the *independent variable*. Independent variables are those quantities that are not being tested. For example, time, soil depth, and altitude are quantities that change, but are not affected by our experiment.

The vertical axis (y), on the other hand, tends to correspond with the *dependent variable*, the variable that changes based on the conditions set out by the independent variable. For instance, the partial pressure of oxygen will change depending on altitude, and nitrogen levels will vary depending on soil depth.

In Figure 1, the altitude is listed on the y-axis and time on the x-axis.

Figure 1



UNITS AND SCALE

We must also pay attention to practical matters like units and scale on each axis.

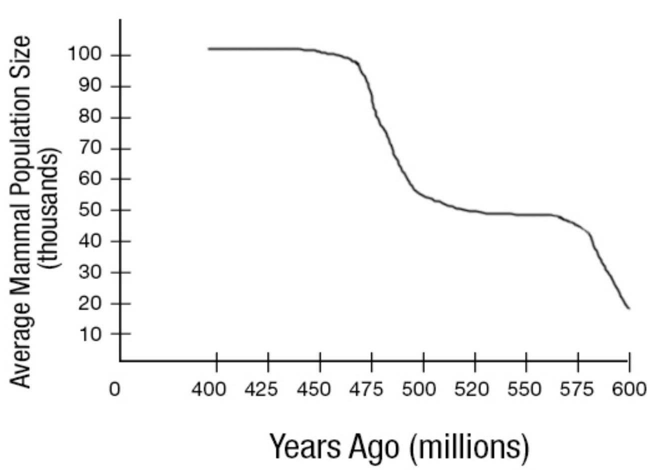
Units are the basic terms by which we count a measurement. Centimeters (cm), hours (h), kilograms (kg), milliliters (mL) are all various units that describe different sorts of measurement (length, time, mass, and volume, respectively, in this case). In Figure 1 above, altitude is in meters (m) and time in seconds (s).

Scale on a graph is the amount of units between each of the tick marks on a particular axis. Sometimes each mark will indicate a single unit change, but they can indicate any change from twenty units to twenty million units (or more!). In Figure 1, the scale of the y-axis is 5, and the scale of the x-axis is 1.

You may need to draw out extensions of the unit marks on the y-axis to help see the values that are indicated at a particular x-coordinate.

Additionally, some graphs may not start at 0 units. You may find a squiggly between the origin and the start of an axis indicating a big jump from 0 to whichever unit is best for the illustration of the data (see Figure 3 below). It's possible nothing interesting happened for the first 40 hours, but then changes occurred in the bacterial colonies each hour for twenty hours on the petri dish you were examining.

Figure 3



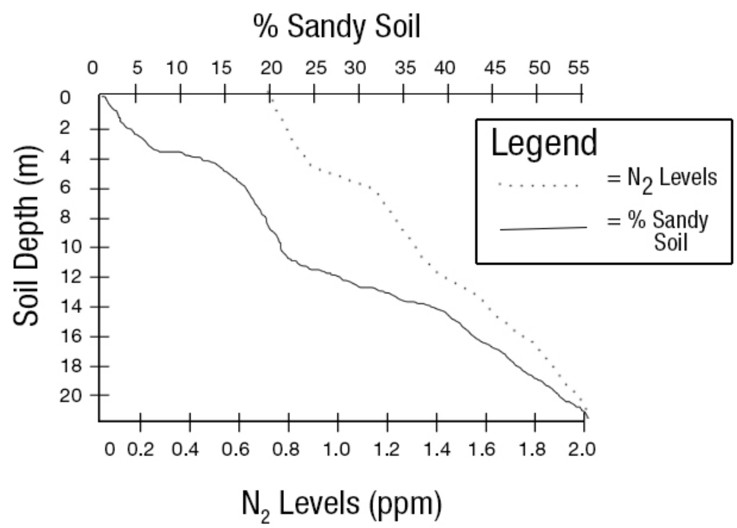
MULTIPLE Y-AXES

You may also find graphs with two y-axes (yes, they can do that). This allows you to plot two separate graphs that both correspond to the same x-axis (like soil depth) but different y-axis values (like nitrogen concentration and percent sandy soil in Figure 2 below).

This allows us to see if trends line up for different quantities and to maybe make a conclusion based on those trends.

For example, if nitrogen concentration decreases with soil depth at a rate of 0.1ppm/m and the percent sandy soil increases a rate of 2.5%/m, they might have similar slopes and even line up pretty closely based on our y-axis scales. That might suggest that nitrogen has a tougher time staying in sandy soil.

Figure 2



On such graphs, it is important to pay attention to legends (boxes that show different patterns) that explain what each type of line indicates (particular year, molecule, animal, soil type, etc.).

SLOPE

Lastly, as we discussed with the nitrogen concentration and sandy soil percentages, it is important to understand *slope*. The slope is equal to the rate of increase or decrease. Negative slopes travel downward from left to right, while positive slopes travel upward in the same direction. Additionally, the steeper the slope, the larger the rate of increase or decrease (speed, temperature, etc.).

TABLES

Just as in graphs, all of the information required to understand the topic is provided in the table; in the case of tables, it's just a matter of identifying what each row and column indicates.

A particular cell (that's what we call a single box in a table) might tell us the number of red cars. We'll know this because it is in the cars row and the red color column. Sometimes groups of columns or rows will have a sort of "superstructure." Cars might be lumped in with vehicles, and red with colors. See Table 2 below.

Table 2

# of Vehicles Owned (millions)				
Vehicle Type	Color			
	Red	Black	Blue	Silver
Car	10	12	8	6
Truck	2	6	3	4
SUV	3	10	4	5
Motorcycle	2	4	1	0.5

It’s important not to get bogged down in specifics on tables. On the SAT, trends are king. To determine trends, we need to devote more of our limited time to finding overall patterns, rather than trying to determine ultra-specific, numerical rates.

For example, in Table 1 below, masses for the nucleus, mitochondria, and ribosomes are about the same for both Samples A and B. Additionally, we could approximate these masses to around 2.5, 4 and 1.5, respectively. These numbers are much easier to work with when doing quick mental math than their specific values are. Additionally, it is acceptable to think of a number like 2.5 for nucleus mass in Samples A and B as one half of the approximately 5 micrograms in Sample C.

We realize this might bug the more precision-minded of you, but we have a very limited amount to time to accomplish sometimes very difficult analytical tasks. These approximations allow us to do so.

In fact, we might notice that Sample C’s values are all approximately double those of Samples A and B. This is likely because Sample C contained the same number of cells, but each of those cells was undergoing mitosis, the process by which a cell makes a copy of itself.

Table 1

Organelle Masses (mg)			
SAMPLE	Organelle		
	Nucleous	Mitochondria	Ribosomes
A	2.4	4.2	1.7
B	2.3	4.4	1.5
C	5.1	8.0	3.1

As mentioned, if trends are king on the SAT Science passages, then exceptions, or outliers, are queen. We must always be on the lookout for particular cells that stand out from the rest.

WRITING & LANGUAGE SECTION

The Writing & Language section makes up the second half of the Evidenced Based Reading and Writing score (scored 200-800).

1) It is always the second section on the test.
2) There are always 4 passages in the section.
3) It always contains 45 questions.
4) It always lasts 35 minutes.
5) There is NO penalty for wrong answers, so make sure you answer every single question even if you have to guess.

Knowing these facts in advance can help you prepare better for test day.

WHAT'S BEING TESTED

This section is essentially a grammar test. As a matter of fact, if you took away the timing element, 90% of the difficulty of the section would be determined by whether or not you know grammar rules. However, that still leaves us with 10% of its difficulty being in other areas. Here are a few rules that will help you master that last 10%.

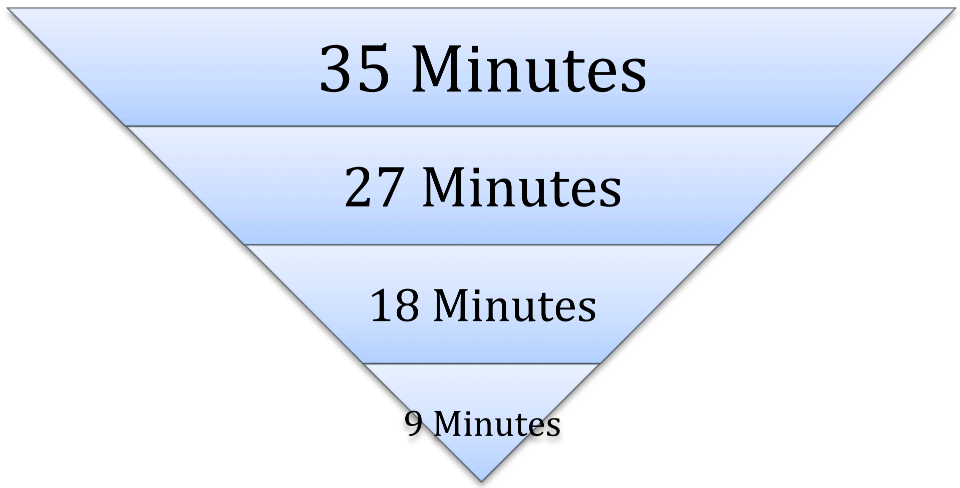
THE 6 TIER ONE TRICKS FOR WRITING & LANGUAGE

1) You have a drop under nine minutes per passage to complete the work.
2) Each question is worth more points than in the Reading section.
3) Passages are broken into small chunks with accompanying questions for easier review.
4) The questions do NOT go in order of difficulty, so try all the questions in the section.
5) If all else fails, and you are going to guess on a question or you don't know how to choose between two answer choices, choose the shortest answer choice.
6) Under no circumstance should you read the entire passage first. Since questions go in chronological order, only read the part of the passage necessary to answer the question at hand.

THE TIER ONE PACING METHOD

As alluded to earlier, we want to make sure that all practice is done timed. As a reminder, here are the rules for your timed practice as you progress through Writing & Language Section prep.

English Section with One Passage Gradation



The idea here is to start from the bottom. You will do all your initial practice with just one passage, allowing yourself approximately nine minutes. You'll do this until you are able to consistently finish on time, then you move on up to two passages in 18 minutes. You'll do that until you consistently finish on time, and then move up again, progressing in this way until you can complete the entire section on time.

Starting from the beginning of your prep, you will want to make sure you do timed practice in this sequence. The goal is not just that you learn how to finish a section on time, but also to gain mastery over your timing so you don't feel pressed for time while completing a section either. This should stop you from making critical careless errors and from experiencing heavy testing anxiety, as well.

THE 4 TIER ONE WRITING & LANGUAGE QUESTION TYPES

As part of the extensive research Tier One does on the exam, we have identified four fundamental question types that make up just about all of the questions you will see on the Writing & Language section on test day. They are: Classic Grammar, Either/Or, Improving Paragraphs, and Vocabulary.

Each of these questions have telltale signs in them that will allow you to instantly know which question type you are working with and there are also unique approaches that can be used to approach these as well.

CLASSIC GRAMMAR

These questions are easy to spot because answer A will always say “NO CHANGE.” Basically, there will be a sentence, or part of a sentence, underlined in the passage. Your job is to choose what the “best” version of that sentence is. Answer choice A is always the option to leave the sentence as is, and then B, C and D will be three alternative choices. These are by far the most common question type on the section.

Here are the three rules for Classic Grammar questions:

1) The best answers are the ones that are most “clear and concise.” If there are two choices that are both grammatically correct but one is shorter (more concise) than the other, the shortest one wins.
2) Unlike in the Reading section, having the context of the previous and next sentences is almost never helpful. So just read the sentence in question.
3) These questions are almost always simple grammar questions. Make sure to utilize the 10 Tier One Grammar Rules below to achieve the requisite mastery you will need for test day.

“EITHER/OR”

These questions are quite prominent and will often show up once per passage. They are identified by the pattern of their answer choices. Nearly 90% of the time the prompt will be a “yes or no” question and there will be two options for “yes” (choices A and B) and two options for “no” (choices C and D). Sometimes there will be alternative language (for instance, asking if a sentence should be kept or deleted, and there are two options for kept and two options for deleted), but essentially it is the same structure of having to choice either one thing or the other.

There are two main steps to successfully answer “Either/Or” questions:

1) First identify which pair of answers makes most sense. For instance, if it is a yes or no question, don’t worry about which choice is the best, simply focus on whether the answer should be either “yes” or “no.”
2) Once you have it down between two choices, your goal is to figure out which answer choice provides the best reason or rationale for that decision. Remember that at its core, the SAT is an exam of evidence and analysis. Therefore, the best answer will be the one that gives the most clearly logical explanation – as opposed to a technical one.

VOCABULARY

Rarely, maybe once per test, you will get a vocabulary question. It will look identical to a Classic Grammar question in that answer choice A is “NO CHANGE,” however, the underlined portion and other answer choices are all single vocabulary words.

The question is asking which of the four vocab words “best completes” the sentence in question. That means there could be two words that technically can work, but the question wants to know the one that is that makes the sentence have the best logical flow possible.

There are four steps to successfully solving a Vocabulary question:

1) Read the sentence while imagining that the underlined word is simply missing.
2) Make a prediction for the type of word, or exact word, that would make sense to fill in the blank.
3) Compare your four choices (the original word as well as the ones in choices B, C and D) to your prediction “conservatively.”
4) Compare your four choices to your prediction “liberally.”

Just remember that the types of words you are going to see are not the most uncommon words out there. They will be relatively “normal” words that maybe have an alternative definition.

IMPROVING PARAGRAPHS

Improving paragraphs questions are essentially testing your ability to alter a paragraph to improve the clarity of the text. These questions are easy to spot because answer choice A does not say “NO CHANGE” and there is no either/or set of choices.

Generally speaking you will see five types of Improving Paragraphs questions:

1) Figure out where the most logical place would be to add a new specific sentence.
2) Combine existing sentences effectively.
3) Move whole paragraphs within a passage.
4) Determine the best/most logical version of a new sentence.
5) Choose the best sentence to accomplish a particular goal (for instance to better prove a certain point being made in the passage).

The overarching goal of these questions is three-fold. First, whatever change you makes needs to be grammatically sound. Second, it needs to be the most concise way to express the ideas in the passage. Finally, it needs to allow for the most logical flow of ideas possible.

TIER ONE’S COMPREHENSIVE GRAMMAR REVIEW

As mentioned above, the primary component being tested on the English section is grammar. Therefore, the best way to prepare for the section is via intensive study and review of the most common grammar rules tested on the SAT.

INTRODUCTION TO PARTS OF SPEECH

When we describe the parts of speech, we'll use the following terms.

- ***subject***
- ***direct object***
- ***indirect object***
- ***object of the preposition***

SUBJECT

A *subject* is the main “actor” in the sentence. It is the word “doing” the verb. Let's look at the sentence:

Jack sent a letter to Diane for me.

Jack is the subject of the sentence; after all, he’s the one doing the sending.

DIRECT OBJECT

A *direct object* is the word being “acted upon.” It is the noun to which the verb is doing the action.

Let's look again at the sentence about Jack above. The direct object is the letter; it’s what is being “sent.”

INDIRECT OBJECT

An *indirect object* is the “recipient” of the results of the action. It almost always is accompanied by the word “to.”

In our example about Jack, the indirect object is “Diane.” She is the recipient of the letter being sent.

OBJECT OF THE PREPOSITION

The *object of the preposition* is the noun that completes the intended meaning of a prepositional phrase. Looking back again at our sample sentence, we see that “me” is the object of the preposition. It completes the meaning of the phrase that started with the word “for.”

PARTS OF SPEECH

PART OF SPEECH	WHAT IT DOES
noun	people, places, things, and ideas
pronoun	replaces a noun
verb	shows action or a way of being
adjective	describes a noun
adverb	describes how a verb is done and MAY end in -ly
preposition	words that show a relationship in space or time, like *by, for*, *to*, *about*, *from*, *beneath*, *in*, *on*, *with*, etc.

NOUNS – SINGULAR, PLURAL, & COLLECTIVE

When a noun represents just one item or person, we call it *singular*. When it represents more than one, we call it *plural*. Plural nouns tend to end in –s or –es.

In the sentence:

Jennifer ate ten cherries.

"Jennifer" is a singular noun. "Cherries" is a plural noun.

Tier One Tip!
Be careful not to confuse a *collective* noun with a plural noun just because the collective noun represents a single group comprised of multiple individuals. For instance, the word “family” is singular, even though it is made up of multiple members of that family. It would need to be “families” to be plural.

PRONOUNS

Nouns can sometimes be replaced with *pronouns*. For instance, the sentence,

Howard likes his eggs with cheese.

could be rewritten as:

He likes them with cheese.

The following chart describes the different pronouns and how they are used:

<table>
<tr><th rowspan="2">Person</th><th rowspan="2">Number</th><th colspan="3">Use</th></tr>
<tr><th>Subject</th><th>Object</th><th>Possession</th></tr>
<tr><td rowspan="2">First</td><td>Singular</td><td>I</td><td>Me</td><td>Mine</td></tr>
<tr><td>Plural</td><td>We</td><td>Us</td><td>Our</td></tr>
<tr><td rowspan="2">Second</td><td>Singular</td><td rowspan="2" colspan="2">You</td><td rowspan="2">Your</td></tr>
<tr><td>Plural</td></tr>
<tr><td rowspan="4">Third</td><td rowspan="3">Singular</td><td>He</td><td>Him</td><td>His</td></tr>
<tr><td>She</td><td colspan="2">Her</td></tr>
<tr><td colspan="2">It</td><td>Its</td></tr>
<tr><td>Plural</td><td>They</td><td>Them</td><td>Their</td></tr>
</table>

Tier One Tip!
Please note that the third person singular possessive pronoun is “its,” not “it’s.” “Its” tells us that something belongs to “it,” while “it’s” is a contraction of “it is,” as in “It’s improper to use ‘it’s’ when you mean ‘its.’”

PRONOUN-ANTECEDENT AGREEMENT

Remember pronouns are used to replace nouns. The antecedent (the word the pronoun replaces) and the pronoun need to match in person and number. We call that *pronoun-antecedent agreement*.

If you replaced the nouns in the sentence:

Eve and Simon went to visit Jason.

You would say:

They went to see him.

"Eve and Simon" would be replaced by "they," the plural, third-person pronoun, and "Jason" would be replaced by "him," the singular, third-person pronoun.

> **Tier One Tip!**
> One of the most common mistakes we see students make in essays, on multiple-choice questions, and in everyday speech is using a third-person plural pronoun when a singular pronoun is called for.

For example, which pronoun would you choose to replace the word "student" in the following sentence?

A student will lead the assembly in saying the Pledge of Allegiance.

It's tempting to say "They will lead the assembly…" but that would be a plural pronoun replacing a singular antecedent. As a result, we need to use two singular pronouns to say, "He or she will lead…" It sounds a bit more awkward, but it is the correct usage.

DON'T CONFUSE PEOPLE WITH A PRONOUN

Sometimes, you can't use a pronoun, because it would confuse readers as to which antecedent the pronoun is referring to. For example, in the sentence:

Grace and Kelly were going to leave on time, but she forgot her purse.

the usage of "she" is incorrect because it is unclear whether we are talking about Grace or Kelly (both common female names).

TOUGH CHOICES (AKA "who" vs. "whom" and "I" vs. "me")

The words "who" and "I" are used as subjects and direct objects, while the words "whom" and "me" are used as indirect objects and objects of prepositions. In the sentence,

Jim and I wrote the congressman.

we used "I" because "Jim and I" is the subject. But, if we followed up with,

The next day, the congressman wrote a response back to Jim and me.

we would use "me" to show that now "Jim and me" serves as the sentence's indirect object.

> **Tier One Tip!**
> If all of this specialized grammar doesn't really float your boat, there are two tricks that will help in most cases.

First, problems associated with "I" vs. "me" almost always occur when one of the two words is used with another noun like in the sentence,

Jim and I wrote the Congressman.

To confirm that we used the correct word ("I" or "me"), all we have to do is eliminate the other noun that has been lumped in with our first person singular pronoun (usually with an "and" or an "or") and see if the sentence still makes sense. In this case "I wrote the Congressman," still makes perfect sense, meaning we used the correct pronoun.

Second, in a sentence posed as a question, we can use a different strategy for "who" and "whom." Simply answer the posed question using either the word "he" or "him." If "he" sounds best, the word "who" should be used in the question, while if "him" rings true, then the word "whom" should be employed.

Whom will I call?

There are two possible replies: either "I will call him," or "I will call he." The former sounds much better, indicating that we should be using "whom." We already do, so we're in the clear.

Practice Questions (these are not SAT questions):

1) <u>Who</u> did you want me to make the check out to, Mr. Underwood?
   a) No change
   b) Whose
   c) Whom
   d) Whoever
   e) What

2) Between you and <u>I</u>, the food here is absolutely awful.
   a) No change
   b) me
   c) myself
   d) mine
   e) it

3) While a soldier must follow orders given to <u>them</u> by a superior officer, an individual citizen need not.
   a) No Change
   b) it
   c) him or her
   d) he or she
   e) it

VERBS & SUBJECTS MUST AGREE

A subject and its verb must always match in number. This is called *subject-verb agreement.* The subject and verb "disagree" when a plural subject is combined with a singular verb, or vice versa. The subject and verb in a particular sentence must also agree in person (first, second or third).

This 2-part method will help you make sure your subject and your verb agree:
1) Identify the subject (a noun) and verb of the independent clause.
2) Confirm that the verb matches the subject in number and person.

Let's look at this example:

Several exotic animals, including a white tiger, <u>has</u> been transported to the zoo in recent weeks.

Notice how the subject of the sentence *seems* to be the "white tiger," but is *actually* the "exotic animals." The verb "has" in this sentence is referring to the subject (which is plural), and therefore "has" (singular) is the wrong conjugation. Instead of being singular, it should be plural like "exotic animals" and therefore be "have."

> **Tier One Tip!**
> The SAT will frequently use the strategy of separating the subject from the verb by using different types of clauses or phrases. It's our job to make sure we can pick out only the relevant information.

Practice Questions: *Identify whether the sentence contains a Subject-Verb Agreement Error, and replace the grammatically incorrect verb (if there is one) with the correct one.*

1. The conspiracy theorists are convinced that the allegation, coming mere hours before the hearing, were planned to sway the public from supporting the lawmaker.

2. Trans fats contained in junk foods have been studied for years, yet only recently has the negative effects become factual.

3. Each of the board members are concerned over the first quarter financial reports.

VERB FORM

A verb's form is best understood as the way in which a verb is conjugated or changed. For instance, the infinitive "to eat" can be conjugated in many different forms; it can be "eating" "ate," "eat," etc. For those of you enrolled in a Romance language in school, this should be a familiar task. You know when to use "soy," instead of "estar," in Spanish or "aller," instead of "allons," in French.

> **The music of the Beatles has managed keeping youths and adults alike entertained for many decades.**

The problem with this sentence is the verb "to keep." The verb is in the wrong form; instead of "keeping" it should say "to keep."

> **Tier One Tip!**
> One of the ways to know if a verb is in the wrong form is to look at the other verbs in the sentence. In the Beatles example, immediately before "keeping," we have the verb "managed" – this can be a strong clue for us. Because the verb "managed" is already conjugated (form "to manage") the next verb should NOT be conjugated. In general we do not conjugate two verbs in a row. Therefore we would know that "keeping" should not be conjugated; it should stay "to keep."

Practice Questions: *Identify any errors in verb form and indicate what would have been the correct verb choice.*

1) After Michael Phelps had swam his way to 8 Olympic gold medals, he was celebrated as the first person to realize the goal.

2) Although the ants were very tiny, each of them was able to carrying many times its own weight in food.

VERB TENSE

Verb tense indicates whether an action takes place in the past, present or future. Tense errors occur when the tense of the overall sentence does not agree with the tense of a particular verb.

> **While all of us watching from the sidelines feared the worst, the injured athlete himself is actually the most optimistic.**

The verbs in this sentence do not match. The verb "feared" is in the past tense while the verb "is" is in the present tense. This should clue you in to the fact that one of them is wrong.

Practice Questions: *Identify any errors in verb tense. Indicate the correct verb choice as necessary.*

1) According to economists at the 2009 panel on free markets, one of the most drastic financial transformations in the 1970's will be the deregulation of banks across the country.

2) Because he is sick when the presentations were given, Shawn is concerned about missing the remainder of the school year.

SENTENCES AND CLAUSES

A *sentence* expresses a complete thought and **must** contain a subject (which can be implied, as "you" is implied in imperative sentences like "Run!") and a *predicate*. A predicate includes a verb and any direct or indirect object it requires. The subject and the predicate together form a core called an *independent clause*. A sentence **can** contain *dependent clauses*, as well, but doesn't always.

TYPE OF CLAUSE	DEFINITION	CAN IT STAND ALONE?
independent clause	The core of a sentence, containing the subject and the verb/predicate, is known as an *independent clause*.	yes
dependent clause	Parts of a sentence that also contain a subject and a verb, but they do not express a complete thought. Dependent clauses are usually set off by particular words like when, while, because, which, who, etc.	no

INDEPENDENT CLAUSES CAN STAND ON THEIR OWN, WHILE DEPENDENT CLAUSES CANNOT

On the chart above, note the similarities and differences between *independent clauses* and *dependent clauses*. Be aware that sentences and independent clauses both may stand alone, but only a sentence can contain more clauses in addition to the independent one. Now, take a look at this sentence:

While he never once took a bribe, the inspector was known to look the other way for friends and family.

"While he never once took a bribe," is a nice turn of phrase, but it can't stand on its own as a sentence. Therefore, it is a dependent clause. However, "the inspector was known to look the other way for friends and family," could be its own sentence, making it an independent clause.

Tier One Tip!
There is a special type of dependent clause that is not signaled by a particular word, but instead is offset by commas. An *appositive* is a phrase that describes the noun immediately preceding it.

In the sentence,

Rosalind Franklin, the woman who first imaged DNA, was not initially credited for her work.

Here, "the woman who first imaged DNA," has a subject and verb and is offset by commas, but it cannot stand on its own, signaling that it is a dependent clause. For our purposes, we will call all appositives dependent clauses.

If a sentence is composed of more than one independent clause, those clauses must be separated by a conjunction ('for,' 'and,' 'but,' 'or,' 'yet,' or 'so') or semi-colon (;). There can be, at most, two independent clauses on either side of a semicolon, meaning a sentence can have anywhere from 1 to 4 independent clauses (excluding any parentheticals) and a nearly infinite number of dependent clauses.

Practice Questions: *Identify a) whether each of the following is a sentence, b) any independent clauses (by underlining), and c) dependent clauses (by circling).*

1. Whether you like it or not, this question won't answer itself.

2. When travelling abroad, I always like to sit next to Gordon, my best friend and college roommate.

3. Because I, the top student in the class, forgot the exam was today.

COMPARATIVE ERRORS

Comparatives are words such as *more* and *most*, or *better* and *best*, that facilitate a comparison between two or more people, objects or activities. Mistakes occur when the wrong version of the comparative is used. There are generally three types of comparative errors.

1. "-est" vs. "-er,"
2. "most" vs. "more
3. double comparatives

"-EST" vs. "-ER"

The lion is the stronger animal in the entire world.

The comparative *stronger* is used incorrectly. We use the suffix "-est" when comparing three or more things; otherwise we say "-er."

MOST vs. MORE

Janice and Regina are similar in many respects, but Regina has the most couth and refinement.

This example should read "more couth" as opposed to "the most couth." The same reason would apply here as it did in the lion example above. "Most" is a comparison for three or more things or people, and here we are only dealing with two people.

DOUBLE COMPARATIVES

A double comparative is when you have two comparatives that are right next to each other.

It has never been more clearer than it is now that the bald eagle must be saved from extinction.

This example should simply read "more clear" or "clearer." "Clearer" is a comparison word and so is "more" – therefore you need to choose only one of them to describe the situation.

Tier One Tips!

1) A comparative should end with "-est" only when comparing three or more things; it will end in "-er" if comparing only two things.
2) You cannot make a double-qualified comparison. If you say "more" or "most" or any qualifier, you cannot have the next word end in "-er" or "-est" as well.
3) "Most" compares three or more things while "more" compares only two.

Practice Questions: *Rewrite the sentence using the correct form of the comparative adjective, using "-est" or "-er."*

1. Of the two performers on stage tonight, Philip is the best dancer.

2. After reading the editor's glowing response to his column, the newspaper writer thought himself the greater writer on staff.

Practice Questions: *Use "most" or "more."*

1. In many ways, John and Brian have similar personalities, but John has the most energy and charisma.

2. As Supreme Commander of the Allied Forces, Dwight D. Eisenhower had the most power than any other general.

Practice Questions: *Can you spot double comparatives?*

1. It has never been more clearer than it is now, just how important a large front line is to a championship contender in the NBA.

2. As one of the most popular spokesmen in the country, Mr. Frindler has never been more busier than he is this month.

ADJECTIVE/ADVERB ERRORS

An adjective/adverb error can occur in only two ways.

ADJECTIVE INSTEAD OF ADVERB

The first way is if an adjective is used instead of an adverb. Here's an example:

On particularly hot days, Bob likes to walk serene along a quite stretch of the beach.

In this sentence, "serene" is the adverb; it describes the verb "to walk." Therefore, "serene" needs to end in an "-ly" like all adverbs – we should use "serenely."

ADVERB INSTEAD OF ADJECTIVE

Adjective/adverb mistakes also happen when an adverb is used incorrectly. To fix this error, you must simply replace the adverb with its corresponding adjective. For example:

The quickly hare far outpaced the tortoise at the start of the race, though his hubris prevented his ultimate victory.

The adverb in this sentence is “quickly” but since it describes a noun (hare) it should instead be an adjective, “quick.”

Tier One Tips!
The simple rule on these problems is to determine if your descriptive word in the sentence is describing a noun or a verb; if it’s describing a verb it should end in “-ly,” if it’s describing a noun it should not.

Practice Questions: *Please circle and correct any misused adjectives and/or adverbs in the following sentences.*

1. Although the new car was the most intelligent designed vehicle that Mr. Johnson had ever driven, it could not prevent him from getting into a fender bender on the highway.

2. The investigation uncovered amazing new proof detailing the rapidly expansion of a new criminal network responsible for the majority of violence downtown.

IDIOM ERRORS

An idiom is a term or phrase whose meaning cannot be determined by its literal definition. In other words, if you take it literally, it doesn’t make sense; rather, the figurative meaning is how to understand it. For example, we say that we listen *to* someone, not listen *at* someone. Also, we say that we have a preoccupation *with* something, not a preoccupation *to* something.

Put another way, language is an agreed upon set of rules to convey a particular meaning. It just so happens that we’ve agreed upon the use of certain words (often prepositions) when used in conjunction with other words. It’s not that there’s a particular reason for the word's use; it’s just what has come to be used over time.

Far away from understanding the true spirit of her beliefs, Joan continued to protest the deficiencies in her life.

The problem here is that it is “far away from” whereas it should simply say “far from.” We might use the phrase “far way from” to describe the distance between to objects, but that is not the intended meaning in this sentence. Remember, the catch is that these errors are simply defined by the way we speak.

The thriller, written from author John Doe, explores the inner psyche of a serial killer.

Once again, the problem here is that we would not say something is “written from” an author; rather, we say “written by.”

Practice Questions: *Circle any idiom errors and indicate the correct word choice.*

1. Extreme preoccupation on success seems to be the only characteristic great business leaders have in common.

2. Among enthusiasts, the baseball card was considered the only one of a kind in the country.

PARALLELISM

Parallelism is most easily defined by discussing the structure of a sentence. The structure of a sentence, as well as the individual points in a sentence, must be similar. The easiest example is a list. If I am giving directions and say, “John has to turn right, then left, then turns right again,” this list lacks conformity. The list says “turn” but then later in the same list it says “turns.” This is a parallelism problem.

Generally there are four different types of parallelism problems on the SAT:

1) **Verbs** - Problems with parallelism and verbs are actually a review of what we did with verb problems above. One of the common verb issues we’ll see is where we have a change in tense or conjugation.
2) **Correlative conjunctions** – These are just examples of words that always get paired together. For instance, “either” always goes with “or” and “neither” goes with “nor.”
3) **Comparisons and antithetical constructions** – Another common occurrence we’ll see of this kind of parallelism error is a sentence that is comparing or contrasting different items. Again, just make sure all the words are in the same form.
4) **Series** – The final place you’ll come across this kind of error is probably the most common. It occurs when you are listing things in a series.

A talented athlete, Bo Jackson has been a baseball player, football player and collaborated on various sports teams.

To make this sentence structure parallel it would have to say he was a “collaborator.” This way it would have noun, noun and noun, as opposed to noun, noun and verb. This sentence is an example of a series error in parallelism.

Another example would be words that always go together, words that always run parallel to each other – for instance, “either” and “or.” If we say “either” in a sentence, it must be matched with “or,” or “neither” and “nor.”

Baseball was popular with Americans before either football and basketball.

The problem here is that whenever you say “either” it must be matched with “or” (so the sentence would say, “….before either football **or** basketball”). This example is a common correlative conjunction problem.

Tier One Tips!

1) Mostly, you will find parallelism problems with lists and comparisons. If you see a list, make sure each item is in the same form. If you see a comparison, make sure the same category of word is being compared (person to person, action to action, etc.)
2) The common correlative conjunctions are the following: either/or, neither/nor, both/and, not only/but also, whether/or, and between/and.

Practice Questions: *Circle any errors in parallelism. Then, correct the error as necessary.*

1) Purchasing fewer toys is one way to save money; to turn off the air conditioner when it is nighttime is another.

2) Each day Jim comes home, he has to turn off the alarm, then turn on the lights, and finally cooks dinner before he rests for the night.

3) The Bullets and the Rancheros may equally popular, but the Bullets are the more talented team and the Rancheros the youngest.

4) The synagogue's program familiarized young people with Jewish history, taught them Hebrew, and they had the chance to learn texts written in ancient times.

COMMAS

Commas are used in many different situations:

1. Begin and/or end a dependent clause, including appositives.
    a. Recall that a dependent clause (with the exception of appositives) needs a verb and subject, meaning a phrase like "who cheat" in

       Students who cheat often lie to their teachers about other matters.

       doesn't use a comma because "who cheat" has no subject other than the main subject of the independent clause.

    b. Do not treat a phrase that starts with "that" as a dependent clause requiring a comma.

       The test that I had previously passed with flying colors now seemed difficult.

2. Introduce prepositional phrases, other introductory phrases (e.g., phrases that begin with 'because,' 'while,' 'if,' 'although,' etc. or single words like 'yes,' 'therefore,' 'however,' 'well,' 'thus,' and 'no').

   During my trip, I plan to dine at a Michelin star restaurant; while it may be expensive, it should be a great experience.

3. Indicate the start of a new independent clause before a conjunction

   I have not yet seen the new Star Wars movie, but I have no doubt that it will be excellent.

4. Separate the items of a list. Lists can be comprised of nouns, verbs, or adjectives.

   When the professor went to the store, he purchased eggs, milk, flour, and butter; the next day, he lectured, graded papers, and attended faculty meetings.

   One of the common traps students fall for is the placement of a comma before the word "and" when "and" is neither being used as a conjunction nor as the indicator of the final item in a list of three or more. For example, in the sentence:

   While she was a great singer, and dancer, she never mastered an instrument.

   There are only two things we are told about her. As a result, we do not need a comma separating "great singer" from "and dancer."

5. Indicate some common sense things like the separation of date from year and city from state or state from country.

   On January 22, 2005, I moved to Los Angeles, California, United States.

6. Indicate direct address (that the person writing or speaking is addressing his or her comment to a particular person/entity). The sentence

   But I really don't want to clean my room Mom!

   requires a comma before "Mom."

7. Indicate the start of a quote.

   William Wallace was as defiant as ever when he shouted, "Freedom!"

8. Separate two adjectives prior to a noun. This can get tricky. The two questions to ask about the adjectives are:
   a. Could you rewrite the sentence with the adjectives in reverse order?
   b. If you put an "and" in between the two adjectives instead of a comma, would the sentence still make sense?

   If you answered "yes" to these questions, then we can use a comma between the adjectives.

   The spoiled, petulant child was difficult for the doctor to treat.

9. Offset an aside (see section on Asides).

PARENTHETICALS/ASIDES

An aside is something the sentence could do without, but maybe adds a little extra flavor. Examine the following sentence:

We went to the mall (finally) to get a gift for Jordan.

The word "finally" doesn't really need to be put into the sentence, but it does add an offhand detail that you want to convey.

We can separate such an aside from the rest of the sentence using one of three different types of punctuation: commas, parentheses, or dashes.

DULL/COMMON/MUNDANE/SIMPLE ASIDES

Examples would be "however," "I think," etc. These always use commas. Such asides must always be sentence fragments. If the aside occurs at the end of a sentence the commas need not open and close the aside.

He met his deadline, sure, but the work was not up to his usually standard.

PARENTHESIS

Use parentheses for a step up from commas in importance or when commas might be confusing. You can have a whole sentence in parentheses, not just fragments. Further, there must always be a pair, one parenthesis that opens the aside and another that closes it—even at the end of a sentence.

I understand that you (the person not doing any of the lifting) want the couch moved, but I don't see why it can't stay where it is.

DASHES

Dashes (—) are the last step up in importance. It should be something you really want to highlight. Like commas, dashes do not require a pair if we are marking off an aside that closes the sentence.

The production—could it have been any longer—felt tedious at times.

COLONS

We use colons (:) in a couple situations.

INDICATE A LIST

The first instance we use a colon is to indicate the start of a list. This usage usually finds the colon coming after words and phrases like "the following," "as follows," and "items." Do not use the colon in this way if it seems as if the sentence could flow without one. You will rarely find colons after verbs; when you do, it is because a complete thought has already been expressed.

To tackle the SAT you will need the following: a willingness to learn, a good work ethic, and an excellent teacher.

DEFINE A TERM

Another time we use colons is to define a term. This usage is similar to an appositive and is a stylistic choice, as it usually ends the sentence.

But here's what he found really interesting: the call came from inside the house.

SEMICOLONS

Semicolons (;) can be used to separate what would otherwise be two sentences that share more of a connection than a period would suggest. On the SAT, you should be aware that the addition of a semicolon can often "fix" a sentence that previously would have been a run-on sentence.

While I am unsure of how this sentence might end, I do know that it can accomplish several things at once; this is not to say that every sentence ought to include a semicolon, for we would not want our readers to become confused or bored while reading a single sentence.

APOSTROPHES

Apostrophes can be used to indicate a contraction, the combination of two words, one of which is usually a verb.

He could've eaten an entire cow; that's how hungry he was.

They can also be used to indicate possession. Please note you need to add both an apostrophe and the letter "s" to a singular word to indicate possession, but only an apostrophe to a word that ends in "s" (including plural words).

Why did you break your sisters' toys, Jimmy?

Practice Questions: *Insert the appropriate (i.e., grammatically correct) punctuation where needed.*

1. Golly Jim I just dont know if I can trust him anymore hes not the same man he once was literally.
2. Its a darn shame that the team once the best in the country could not win a single game.
3. The fastest may reach the finish line first but the real fun is in the chase then again I've never been the fastest in the bunch.
4. Oddly enough she could never remember the date of their anniversary December 16 1941 which was also the birthday of their first child.
5. The dog that protects its master is worthy of praise which is only fair.
6. The Italians produced the best fresco work during the Renaissance and Baroque eras and the French created the finest bronzes.

SUPERHEROES DO GOOD. UNLESS YOU'RE VOLUNTEERING AT A SOUP KITCHEN, YOU DO WELL. (AND OTHER COMMON MISCONCEPTIONS)

The following list should be thoroughly understood.

1. *Good vs. Well.* "Good" can either be used as a noun or an adjective. When people do "good," they are doing something morally upright. If they are performing the action skillfully, efficiently, or effectively, they would be performing it "well." "Well" is generally an adverb.
2. *Lie vs. Lay.* Let's first say that the meaning of "lie" related to any falsehoods is fairly simple. What we're concerned with is the usage regarding setting things down or assuming the position most people prefer for sleep. When used in the present tense, the word "lie" would get used to indicate reclining, as in, "I lie down on the bed before sleep," while "lay" would be used to tell the reader/listener that something is being set down, as in "You can just lay the phone down on the counter." Where it might get confusing is in the past tense. "Lay" is the past tense of "lie," while "laid" is the past tense of "lay."
3. *Who vs. Which.* "Who" is used to start dependent clauses related to people, while "which" starts such clauses related to things, animals, or concepts.

4. *Less vs. Fewer.* We use "fewer" to indicate a smaller number of quantifiable things (things I could count), people, etc. "Less" gets used for statements indicating a smaller extent for non-quantifiable concepts.

There are fewer people taking the exam than I would have expected, but the exam itself was less difficult than I thought it would be.

5. *Between vs. Among*. There is a common misconception that "between" cannot be used for more a list of three or more words, while "among" can. The real difference is that "between" indicates a certain distinctness in the items/people/groups listed, while "among" may be used for less distinct separations. You can choose between dogs and cats and the pet store, or you can choose among all the animals.
6. *Led vs. Lead.* "Lead" can either be pronounced such that it rhymes with Ed or seed. In the former case, we mean the metal, and in the latter case, we mean the verb that describes what a leader does. "Led" is the past tense of that verb, not "lead."

The leader of the lead miners union led his fellow workers in a strike.

7. *To vs. Too vs. Two.* "To" is used as a preposition or to begin an infinitive. "Too" is a synonym for "also" or "to an excessively great extent", while "Two" is the spelling of the number 2.

To say that the two of them were inseparable would be too much of an understatement.

8. *There vs. Their vs. They're.* "There" indicates a place (figurative or literal), "Their" is the third-person possessive plural pronoun (meaning "it belongs to them"), and "They're" is the contraction of "they are."

They're going to ride their sled down that hill over there.

9. *It's vs. Its*. We have already discussed this, but to be absolutely clear here is one more example.

It's hardly surprising that the dog refused to give up its bone.

10. *Affect vs. Effect*. The word "affect" is always a verb and essentially means "to have an effect on." As a noun, "effect" is simply a consequence or result. As a verb, "effect" could be understood to mean "bring about" or "cause."

He affected everyone around him in such a positive way, while she promised to effect change in the political process.

11. *Principle vs. Principal.* Principles tend to be truths or laws (e.g., the principle of cause and effect). Something that is principal, however, is of primary importance (e.g., the principal soloist in the orchestra). The noun version of the word principal tends to mean the head of a school.

12. *Further vs. Farther*. "Farther" denotes distance, while "further" denotes extent

I travelled farther than my brother did to attend the party, but any further discussion of the matter would be unproductive

Practice Questions: *Circle any incorrect SAT word choice errors and indicate the correct replacement word.*

1. As he lay down on the sofa, his principle concern remained the family's finances.

2. Its truly amazing what an effect weight loss can have on the heart.

3. While my parking space may be further away from the building, I know that it is parked among less cars and is less likely to suffer from dents and scratches caused by careless drivers.

OTHER CONCERNS

Sometimes there is more at play in a question than simply grammar. Let's take a moment to discuss the factors that can affect your answer choice.

AUTHOR'S TONE

If the author of a passage is writing formally, you should not choose an answer choice that uses personal language (e.g., "you," "me," "our," etc.). Likewise, if the tone seems to be more informal, and the vocabulary is a bit simpler, you wouldn't expect the word you choose to be a tough vocabulary word. That being said, we cannot sacrifice the author's original meaning simply to elevate the vocabulary.

REDUNDANCY

The SAT tests on repetitive word choice.

The store annually donated more than 1 ton of food each year.

The words "annually" and "each year" mean the same thing, and the sentence needs only one of those expressions.

DANGLERS

Danglers happen when the subject of one clause does not agree with the subject of the main clause of a sentence. If they do not agree, the first clause is said to "dangle." In other words, if your sentence starts with a dependent clause which itself starts with an –ing word, then the first noun that shows up after the comma must be what the –ing word is describing.

Looking up from the pool, the diving board appeared much taller than it really was.

The first clause, "looking up from the pool," needs to be describing the same subject as the main clause, "the diving board etc." The problem in this sentence is that the main subject of the sentence is "diving board" but the verb in the first clause is "looking." Therefore it seems like the diving board was looking up from the pool! Obviously that is a problem. A quick change to "When viewed" instead of "Looking up" fixes all of our problems.

ACTIVE vs. PASSIVE VOICE

Active voice is the standard, most "lifelike" way of presenting a thought. A simple example of this is the sentence,
The boy threw the ball.

Passive voice changes the emphasis. Instead of the "actor" or "doer" of the verb being the subject, we switch things around so that the thing that is "acted upon" is the subject. For instance,

The ball was thrown by the boy.

would be one such sentence. For the most part, the SAT prefers active voice. Be careful not to pick an answer choice that is in passive voice, unless the author has previously used the style or the portion of the sentence that is not underlined forces you to use passive voice. All of this is not to say that passive voice cannot be used effectively in your own writing. It can, but those cases are rare.

Practice Questions:

1. In actuality, the <u>real</u> levels of carbon monoxide have increased since we removed the fireplace.
    a. OMIT
    b. Actual
    c. Theoretical
    d. Authentic

2. Searching frantically for assistance, <u>John's suitcase was misplaced</u>.
    a. No Change
    b. the suitcase was misplaced by John.
    c. John misplaced his suitcase.
    d. a misplacing of the suitcase occurred due to John.

3. Throughout most of the 19th and early 20th centuries, African Americans <u>were disenfranchised by</u> most states in the Deep South.
    a. No change
    b. disenfranchised
    c. was disenfranchised by
    d. were caused to be disenfranchised by

4. [2] The mice reserved for experimentation were all treated with the utmost care. [3] The same cannot be said for those rodents outside the lab, who frequently <u>keeled over</u> after consuming poison laid out by the groundskeeper.
    a. No Change
    b. expired
    c. suffered
    d. bit the dust

HIGHER-LEVEL QUESTIONS

Macro-level (sentence, paragraph, and even passage level) questions can be tough, but if we break the questions down into their component pieces, while keeping track of the overall passage flow/meaning while doing our initial read-through, the process becomes a bit easier.

MOVING SENTENCES

There are two basic questions types in this category:

1. Choosing the insertion spot for new information ("The author is considering adding the following…After which of the following sentences would this new information best contribute to the paragraph")

2. Finding a new spot for pre-existing sentences ("The author is considering moving Sentence 3. In the context of the paragraph, after which of the following sentences would the sentence add most to the logic of the paragraph.")

Besides more abstract concepts like overall paragraph flow, there are a few tricks for these questions.

Tier One Tips!
For both types of questions, pronouns can be a huge help. Identifying the antecedents for words like "this," "those," etc. can help us to locate the position our new or old sentence should be placed in the paragraph. There is an unspoken rule in the English language that those words which modify other words should be placed as close to each other as possible with minor exceptions for stylistic flourishes. The same is true for pronouns. As we move too far away from the antecedent, it becomes unclear what the pronoun is referring back to.

Particularly in the first question type (new information), the transition included in the new sentence can play a huge role. Words like "therefore" and "thus" indicate the conclusion of something that could be stated as an argument. The words that come after the "thus" or "therefore" must be proven by the previous sentence or sentences. The opposite is true for words like "although" and "however." The words following "although" and "however" in the sentence must contradict the previous sentence's claims.

Practice Questions:

[1] In the quiet village of Smytheton, a young boy travelled down an empty road. [2] No one else was awake. [3] The silence seemed almost oppressive, as if a great weight was pressing the citizens into their beds, preventing them from rising and the town from stirring. [4] Yet, he walked alone. [5] The boy seemed to mumbling one side of a conversation, pausing briefly every once in a while to listen and nod. [6] After reaching the city square, the boy opened his previously cupped hands from which flew the largest woodpecker anyone in the town, had they been awake, would have ever seen. [7] The bird flew straight at the magnificent bell atop a pillar at the town's very center, and when its beak struck the bronze, it was as if all the bell's in the whole wide world rang at once. [8] Slowly, at the speed of a cat stretching its frame before exploring finer pastures, people began to wake. [9] By the time the hustle and bustle of market day arrived, the boy had gone. [10] The only sign he had ever visited was a small dent, one amongst hundred (maybe thousands) in the side of the bell everyone took for granted.

1. The author is considering making a change to Sentence 2. Which of the following options produces the most logical change in the passage?
    a. No Change
    b. Omit Sentence 2 altogether
    c. Adding "Yet" to the beginning of Sentence 2
    d. Adding "For now," to the beginning of Sentence 2

2. The author wishes to move Sentence 4. Where should the sentence be relocated?
    a. Where it is now.
    b. After Sentence 1
    c. After Sentence 5
    d. After Sentence 6

3. The author wishes to describe the daily life of an average resident of Smytheton. Did the author accomplish this goal?
    a. Yes, the author describes an event that is implied to take place each day.
    b. Yes, the author describes the daily life of the young boy.
    c. No, the author fails to describe the rest of the boy's day.
    d. No, the author focuses on the boy, who may or may not be a resident of Smytheton.

MATH SECTION

The Math is scored from 200-800 points and represents 50% of your overall SAT score. It is also broken into two sections – a 20-question, 25-minute "No Calculator" section; and a 38-question, 55-minute "Calculator" section.

CALCULATOR VS NO CALCULATOR

The first Math section you will find comes in section 3 and there are no calculators allowed. You will be given 15 multiple choice questions followed by five "grid-in" questions where you will need to come up with the answer on your own and bubble in the numbers. Every one of the 20 questions can be answered within the allotted time without use of a calculator – your job is simply to think critically and utilize the resources taught in this chapter to get through the information.

The second Math section you will find comes in section 4 and here you are allowed to, and encouraged to, use your calculator. This section has nearly twice as many questions and is therefore worth nearly twice as many points, making it a much more "important" section to your overall score. The first 30 questions will be multiple choice and the final eight questions are "grid-ins."

GRID-INS

Of the 58 total math questions on the exam, 13 of them are grid-ins (otherwise known as student produced response). There is essentially no difference to how you solve these questions other than the fact that guessing is much harder. Instead of simply choosing from four answer choices, you are actually solving for the correct answer and bubbling it into a little grid on the exam.

ORDER OF DIFFICULTY

The Math generally "trends in difficulty," meaning that questions in the beginning of a section or generally easier than later in the section. However, there is one exception: the grid-ins.

An easier way to view the order of difficulty would be to say that multiple choice questions generally go from easier to harder and then when you get to the grid-ins. At that point, the difficulty level starts over, once again going from easier to harder. In that way, question 31 on the calculator section will be much easier usually than number 25 even though it is later in the section. That's because question 31 is the first of the grid-ins.

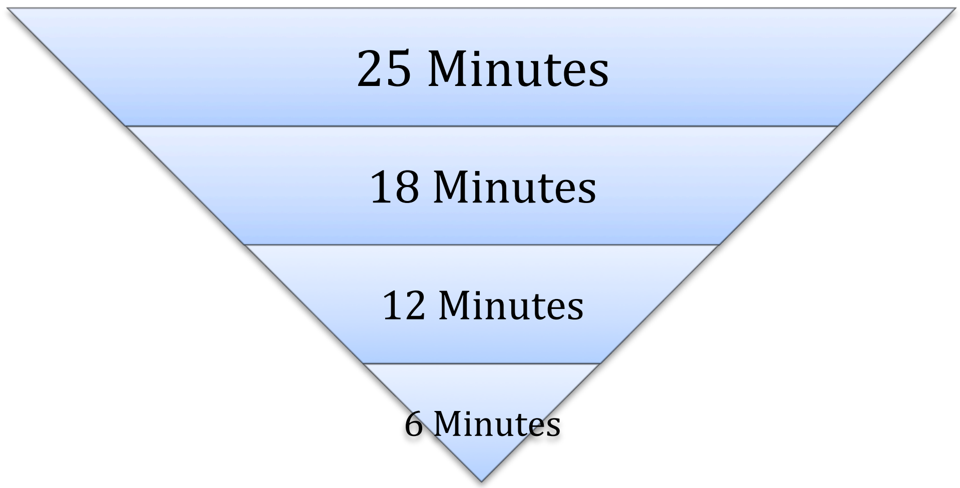
WHAT'S BEING TESTED

The Math section is primarily a comprehensive exam of the last five or so years of math topics you have covered in school. Therefore, knowing all math subjects from arithmetic through trigonometry is critical. However, there are a few other tricks that are really helpful for this section.

THE TIER ONE PACING METHOD

As alluded to earlier, we want to make sure that all practice is done timed. As a reminder, here are the rules by which you should time your practice as you progress through your exam prep.

Math – No Calculator Section with Question Gradation



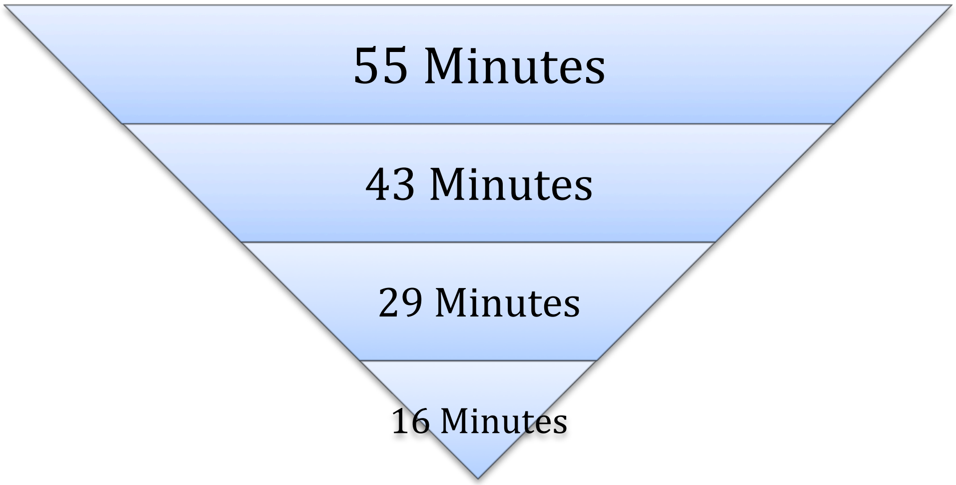
First we will start with the "No Calculator" section. The idea here is to start from the bottom. Unlike the other sections which have passages, the Math section simply has questions. Therefore, in the above graphic, we are going in groups of questions until we hit our total of 20 questions in the section

The one catch is that the section starts off easier and gets harder within the first 15 questions (the multiple choice questions). Therefore, here is the rule:

1) The first six minutes should be used for the first seven questions.
2) The next six minutes should be used for five questions.
3) The next six minutes should be used for three questions only, as these are the hardest multiple choice ones.
4) Finally, use the last seven minutes for the five grid-ins.

Just like the other sections, you are going to start off with only the first seven questions done in six minutes until you master them. When you are able to consistently finish on time, then you'll move up to 12 questions in 12 minutes. You'll do that until you consistently finish *those* on time, and then move up again, etc.

Math – With Calculator Section with Question Gradation



For the second Math section (use of calculator permitted), there are 38 questions and 55 total minutes allowed. In the above graphic we have made it "bottom heavy." Here is how it will work:

1) The first 16 minutes you will do 15 questions as these are the "easier" ones.
2) The next 14 minutes you will do nine questions.
3) The next 13 minutes you will do six questions, as they are the hardest.
4) Finally you will do the eight grid-ins within the final 12 minutes.

TIPS AND TRICKS

The Math sections are not *all* about math knowledge. There are plenty of tricks, tips, and methods that can play a role in increasing your score.

SKIPPING AND SPECIAL PACING

A key difference between Math and the other sections is that the Math sections (as explained in depth above) generally go in order of difficulty. Practically speaking, that can make timing and pacing particularly tricky because as you get deeper in a section, the questions will likely take longer to complete.

Part of having a smart method for the Math on the SAT is knowing how to skip and pace. Essentially, once you hit harder questions, it's important to skip them and come back to them at the end if you have time. Remember, the section generally gets harder, but that doesn't mean each successive question is always going to be harder for you personally. It's very likely that some later questions will be very easy for you and some won't. Make sure to spend no more than 10-20 seconds on these later questions, making a quick judgment about your ability to solve them.

Further, don't forget that the level of difficulty "resets" once you hit the grid-ins. That is a valuable thing to remember when spending time on questions – make sure to leave ample time to get to the easier questions.

TRANSLATING ENGLISH INTO MATH

Variables aren't the only letters we see in math problems on the SAT. Very frequently, we will be required to translate a situation expressed in words into mathematical operations to solve a problem. The following table will prove useful. Whenever you see one of the words in the column on the left, it tends to correspond to the mathematical operation or term on the right.

English	Math
Plus, more, increased, total, add, sum	+
Minus, less, fewer, decreased, subtract, difference	–
Times, of, multiply, product	× (multiplication sign, not the letter x)
Divided by, quotient, for, per	÷
Is, was, equates, equivalent, equals	=
A number, an integer	n, p, x, y, z, etc.
Consecutive Integers	x, x+1, x+2, x+3. …
Consecutive Even or Odd Integers	x, x+2, x+4, x+6, …

Practice Questions:

1) If eighteen is four less than twice a number, what is that number?

2) If five from a number equates to 200% of product of seventeen and that same number, how much is that number?

READING CHARTS & TABLES

Before diving into the charts and graphs, we need to first discuss a few terms that may come up in the course of your evaluation of these figures. Charts and graphs are ways of representing data. The study of data analysis is known as statistics. Those of you taking or have taken statistics may recognize these terms.

Standard Deviation

We don't need to discuss the formula involved with standard deviation for the SAT, so much as understand the basic concept. In any study or survey, we have an average, or mean. The more data points/number of respondents to the survey that are farther away from the mean, the greater the standard deviation. They *deviate* from the mean.

Margin of Error

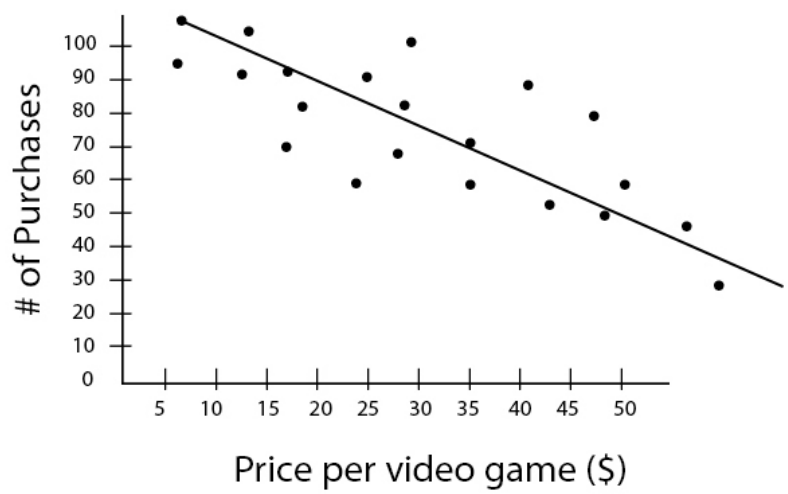
A margin of error is used when reporting the mean of a survey or study because we can never be certain unless we sample every single individual in a population. That's just not feasible. Margin of error must be accompanied by a confidence interval, which will always be 95% confidence interval on SAT. This means that we are 95% certain that the true mean will be plus or minus the margin of error from the mean. The confidence interval can be affected by both sample size and standard deviation. Bigger sample size means we can get closer to reality, so the margin decreases. Wider standard deviation means we're less certain about a mean so the margin increases.

Validity of Sampling

Must be a random sample (otherwise there's a bias) and if we're looking at the efficacy of a treatment (how well people respond to a drug, how much a tree grows when a sugar solution is added to the soil, etc.) we need to randomly apply the treatment (some trees get regular water and others get the sugar solution, but we randomly choose which trees get which).

Now let's get into some of the charts and graphs you might see on the SAT.

Scatterplots



In the above graph we see a series of dots indicating individual data points. Some graphs may show the solid line, as drawn in the graph above. This solid line is what we call the line of best fit, meaning it represents the average positions of these points as we move across the graph. This line can be translated into a linear function by determining a slope and plugging one of the points it passes through into the point-slope form. The line of best fit, then allows us to predict where future points might be.

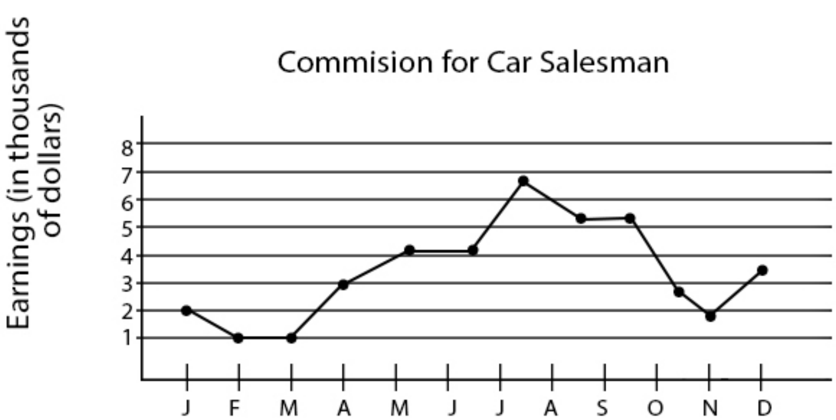
You may be asked to predict a point further along the x-axis (perhaps a $55 price on this graph). If the line of best fit is not drawn, doing so yourself allows you to solve for this value.

Table 1

Time (years)	Tree Height (ft.)
5	6
10	8
15	10
20	12
25	14

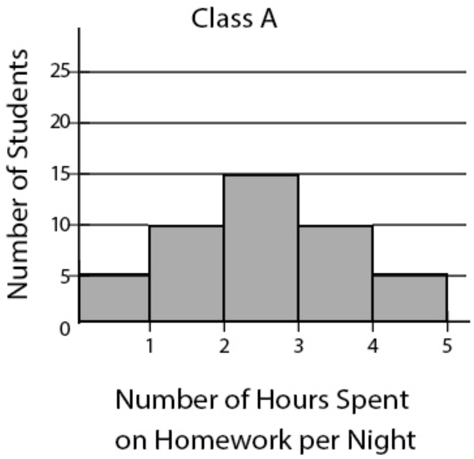
Most times, when presented with a table like the one above, you will be asked to find the function that matches the data. Just as in determining a future data point in a scatterplot, you will need to use the point-slope form to solve for the function at hand. Use any two sets of values (with one column treated as x's and the other as y's) to find the slope and then simply plug in another point into the point-slope form.

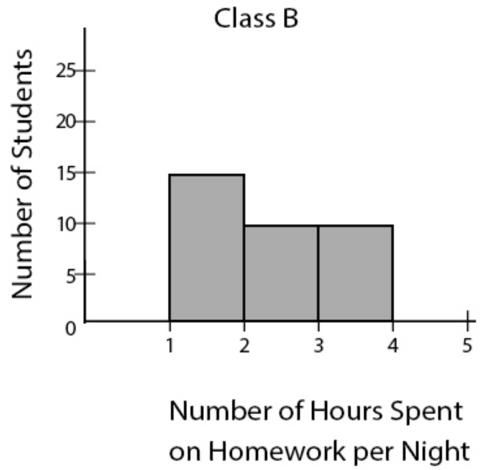
Line Graph



Line graphs are fairly simple to follow, and most questions will ask for identification of trends. The general trend in most lines is what we call a slope! The steeper the slope, the greater the rate of change. For example, you might be asked regarding the above table, between which two months does the data show the greatest rate of decrease? October to November clearly shows the steepest line in the negative (from right to left downward) direction, though we can also confirm this by calculating a slope.

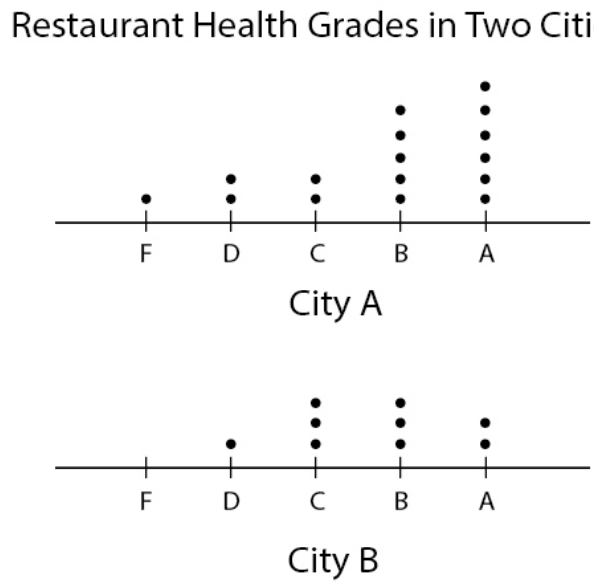
Bar Graph / Histogram





Bar graphs can be fairly simple to read, although you may not be familiar with the histograms as shown above. Histograms allow us to represent data without needing a scatterplot. In the histogram for Class A, 5 students studied between 0 and 1 hour each night. That could mean 4 studied 0.75 hours and 1 studied 0 or any combination of 5 students and times less than one hour. A bar graph is better used when we have precise responses (for example, a survey taken that only allowed students to choose 0, 1, 2, 3, 4, or 5 hours), as the bar should be placed with the x-axis value immediately beneath it.

Dot Plot



A dot plot is a very simple way of representing data. Each dot, unless otherwise specified, represents one response/result in a survey/experiment. For example, in the above graph, the two dots above the letter C in the plot for City A indicate that 2 restaurants received that grade. We can analyze these graphs by finding mean, median, and mode as well as a general impression of standard deviation.

Practice Questions

1) In the table shown in this chapter, find the function that describes the data.

2) In the histogram in this chapter, which Class shows the greater deviation?

3) In the dot plot in this chapter, what is/are the mode/modes grade/grades for City A? For City B?

NATURAL REACTIONS

There are a number of questions on the Math section of the SAT that fall in to one of two question types that we will describe below. Although, generally speaking, this section is difficult because of a combination of complicated math and limited time allotted to solve problems, there is another concern: *Almost all the questions on this section are word problems*.

The main difference between an equation and a word problem is really quite simple: with most equations, it is readily apparent which math concept you need to apply to the problem. The issue is simply doing the math well. In a word problem, however, the main issue you may have is answering the question, "What in the world do they want from me anyway?!"

The method we implement to help solve some of these complicated word problems is something we call "natural reactions." The idea of "natural reactions" is to help eliminate a lot of the stressful and complicated thought processes that you usually use when approaching a new problem.

The trick here is to recognize certain clues and keys in the question that will help you to have a simple and quick reaction to them. For instance, if we could give you a straightforward rule that almost every time you see the number "4" on the test, the answer is "B," wouldn't that be amazing!? Obviously this isn't true, but the point is that if you can have simple rules of how to react to key stimuli in a problem, your life will be much easier. It would allow you to get over the question of, "What math do they want from me?" and let you showcase your other talents.

CHOOSING NUMBERS

Whenever you see variables (usually x or sometimes y, z, etc.) in your question *and* answer choices, your first, quick and natural reaction is to pick simple numbers to replace those variables.

This method is wonderful when it works, and is applicable often enough that you should ALWAYS try it if the above scenario presents itself. However, please note that it does not always work. There are questions on the ACT that have a variable in both the question and the answer choice where this method will simply not lead to the right answer. But don't let that stop you from trying. When this works, it works like a dream!

In terms of implementing the method, your first goal is to try to pick the simplest numbers possible. Keep it single digit if possible (avoiding 0 and 1), keep it positive and try to choose whole numbers (no fractions or decimals).

Also, you must follow all the rules laid out in the problem. Often, there will be limitations given in the problem about the types or size of numbers you can choose. Other times, you will see limitations in terms of percentages (meaning you can only choose 1-100), or time (0-60 seconds or minutes), etc.

Tier One Method for Choosing Numbers

1. Replace the variable(s) in the problem with a simple number that follows the rules of the prompt.
2. Solve the problem.
3. Plug the number you chose in to your answer choices, looking for an answer that matches the solution you found in step two.

Practice Questions:

1) There are g gallons of oil available to a particular gas station on any given day. If x gallons were used by 5:00 p.m., then, in terms of g and x, what percent of the oil was not used?

   A) (100g)/x %
   B) g/(100x) %
   C) (100(g – x))/g %
   D) (100(x – g))/g %
   E) (100(g – x))/x %

2) John signed up x new members on Day 1, 4 times as many members on Day 2 as on Day 1, and 3 more members on Day 3 than on Day 1. What is the average (mean) number of members he signed up per day over the last 3 days?

   A) 2x
   B) 2x + 1
   C) 2x – 1
   D) 3x + 4
   E) 4x – 4

3) At an art sale, a consumer can buy a single painting for Y dollars. Each additional painting the consumer purchases costs 50% of the value of the first painting. Which of the following represents the consumer's cost for X paintings purchased?

   A) x(y+1) / 2
   B) y-x / 2
   C) y-x-1
   D) y+y(x-1) / 2
   E) y+ (x-1)/2y

Tier One Tips!

1. If you are doing a problem revolving around percentages, choosing 100 is often the best number to start with!
2. There are a couple exceptional cases where you can choose numbers even when there is a variable in the question but NOT the answer choice. That is usually only the case when the problem asks you to figure out the total number of possibilities for something being true.

BACKSOLVING

Backsolving is very similar to Choosing Numbers. The main difference is that instead of making up a number to replace our variable, this time we use our answer choices.

Whenever you see variables in your question but NOT your answer choices, your first, quick and natural reaction is to plug your answer choices into the variable in the problem.

Also, just like with Choosing Numbers where we mentioned that although this method does not work 100% of the time, nevertheless it is a fantastic resource that should be used consistently.

1. If x and y are positive integers and $x^2 - y^2 = 11$, what is the value of x?

   a. 4
   b. 5
   c. 6
   d. 7
   e. 8

2. Elite Refineries Co. purchased a piece of mechanical equipment for \$12,000. The value of the equipment depreciates by 20 % each year. The value, in dollars, of the equipment *n* years from the date of purchase is given by the function V, where $V(n) = 12000(4/5)^n$. How many years from the date of purchase will the value of the equipment be \$6,144?

   a. 1
   b. 2
   c. 3
   d. 4
   e. 5

3. If $|x^2-7|<x$, which of the following is a possible value of x?

   a) 1
   b) 2
   c) 3
   d) 4
   e) 5

Tier One Tips!

1. If the problem asks you to find the "greatest" possible value, start off with the largest answer choice; if it asks for the "smallest" possible value, start off with the smallest answer choice.

2. If you are running out of time, you can use this method to get rid of incorrect answer choices. Remember, you should be answering every question even if it means you have to guess – so whenever you can eliminate answer choices to increase your likelihood of getting it right, that is a wonderful resource.

REPRESENTATION IN CONTEXT

An interesting question type on the exam is what we call Representation in Context. Essentially, you will have a normal word problem that will end in a question asking what a particular number or variable means or represents or even what the best interpretation is in this particular problem.

Here is a real example:

A landscaping company estimates the price of a job, in dollars, using the expression $60 + 12nh$, where n is the number of landscapers who will be working and h is the total number of hours the job will take using n landscapers. Which of the following is the best interpretation of the number 12 in the expression?

A) The company charges $12 per hour for each landscaper
B) A minimum of 12 landscapers will work on each job.
C) The price of every job increases by $12 every hour.
D) Each landscaper works 12 hours a day.

Generally speaking, the most effective way to find a solution is to utilize Choosing Numbers. By eliminating the confusing variables, you will be able to better understand the context of the word problem and the accompanying equation. Once you have this clarity, you are simply finding an answer choice that matches the facts of the problem.

CALCULATOR TIPS & TRICKS

Some Brief Advice

First, check to see if your calculator is on the College Board's approved list (found here https://collegereadiness.collegeboard.org/sat/taking-the-test/calculator-policy).

Second, this guide is primarily written for owners of the TI-83, TI-83 Plus, and the TI-84 in mind. These are some of the most common entry level graphing calculators on the market are readily available both in stores and online (I actually recommend eBay as a decent place to pick up a used one).

Third, you should always do your homework and practice tests with the calculator you plan on using on exam day. The fewer changes we have between practice and test day, the better.

Fourth, (and I cannot stress this enough), make sure your calculator has fresh batteries the day before you take the exam. It would be absolutely demoralizing for you and your instructors for all of the preparation you have done to be marred by having your calculator fail you on exam day.

Back to Your Regularly Scheduled Programming

You may hear some teachers say that the calculator is a crutch. That you really ought to be able to do all of this in your head or on paper, just in case. While that may be true off the battlefield that is the SAT, we're only interested in how to best to beat this test. The calculator can be a valuable weapon in this fight, but like any tool, if we don't know how to best use it, it can do more harm (like wasting precious time) than good.

Try to incorporate a few of these techniques into your problem-solving regimen. You may find they take longer at first, as you learn them, but in the end, they will frequently save time.

With any operation on the calculator, we need to make sure we are paying attention to the order of operations. Your calculator will follow PEMDAS to the letter, meaning that if you enter 3*2+4, it will spit out 10. The problem is that you may have meant 3*(2+4), which is an entirely different number, 18. The most common instance we see students make mistakes like this is while entering complicated division problems. $\frac{2 + \frac{1}{2} * 3}{3/5}$ might seem simple on the page, but in our calculate is has to look like this: (2+(1/2)*3)/(3/5). If you forget a parenthesis sign (remember, they always come in pairs) your calculator won't spit out an answer, instead it will say ERROR: SYNTAX. Also, if we neglect to put a pair of parentheses around the numerator, the order of operations will change the answer entirely by dividing (1/2)*3/(3/5) before adding 2.

Speaking of order of operations, we can easily do complicated exponents in our calculators by using the ^ symbol on the calculator. $8^{2/3}$ would be written as 8^(2/3). Notice the parentheses. If you simply want to square a number, all you have to do is press the x^2 button on the calculator, however. The reason we teach the process is because you will sometimes see variables to fractional powers, and then your calculator isn't of much help at all, unless you use the Choosing Numbers Natural Reaction. With roots, there is a standard square root button reached by hitting the 2nd or Shift key followed by the x^2 button. What about more complicated roots, like the fifth root. While we can do it directly by hitting the number of the root we want to take (in the fifth root, we would type 5), MATH, and then scrolling to a button that looks like a standard square root symbol with an x jammed in it, and hitting Enter. Finally, we type the number we're actually rooting and hit ENTER. If that seems too complicated, jut remember that roots are simply a fraction power. The fifth root is just the number to the 1/5 power. And we remember how to do fractional powers right?

One quick tip on finding radians in terms of π. If your calculator spits out a radian value that just looks like a regular number, simply divide that number by π and convert the answer to a fraction by hitting MATH, making sure you're in the NUM column by hitting the right cursor arrow, and choosing the entry labeled ▶ FRAC. The resulting fraction is the coefficient on π.

Calculators can also be great at solving problems relating to functions. If you ever want to find the x-intercepts, maxima, minima, and intersections, simply plug in the function(s) into the Y=, making sure you use correct parentheses, and hit 2nd and then TABLE to calculate whichever value you wish. When calculating intercepts, it will ask you for a LEFT, RIGHT, and a GUESS. Simply make sure that the arrow, which will move left or right as you press the left and right buttons, is to the left of the intercept you want to identify before you hit ENTER when it asks for LEFT. It will then ask for RIGHT. Move the cursor until it is to the right of the intercept before hitting enter. You can then hit enter again at GUESS (no function you'll be entering for this exam is complicated enough to where you actually need a real guess; the computational power of your calculator is sufficient to the task). On the bottom left of the screen, you will now see a value has been spit out. It's the same process for finding maxima and minima. For Intersections, it will ask for the first function and the second function. The left and right arrows on your calculator will allow you to switch between the two. You'll also be prompted for a GUESS, which you can just hit ENTER at to avoid any confusion. The point will then be highlighted and displayed.

Using the Intercepts function can be quite useful when factoring. All you have to do is enter the quadratic you're examining as a graph in the Y= tab and use the process listed above to find the "zeroes."

Intersections can also be useful to solve equations. Simply set one function (y_1) as the left hand side of the equation, and set another function (y_2) as the right hand side of the equation. Find the x-coordinate of their intersection point, and you've found the solution! You can even find the two solutions for absolute value equations, so long as you remember you can input an absolute value into the function by hitting MATH, moving right to the NUM column, and selecting abs() before you enter the function. Make sure to close the parentheses at the end! It's like closing the absolute value bars.

Additionally, you can also input various programs into your calculator that will help you to solve certain problems. For example, there are programs that would allow you to input the coefficients a, b, and c in a quadratic of form ax^2+bx+c and have it spit out the solutions to the Quadratic Formula. The University of Arizona Math Department as a good page on that here (http://math.arizona.edu/~krawczyk/Calculator/TI83PLUS/TI83PQF.html).

Finally, it can sometimes be difficult or time-consuming to find the mean or median for a list of numbers. We can remedy that by using our calculator. First hit STAT, then select 1: EDIT…
This will present you with a something that looks like a table. Enter the 1^{st} value we have, and then hit enter. Repeat until you have input all of the values in your data set. Now press 2^{nd}, then STAT. Move the cursor to the right twice until you're under the NUM column. You should see mean and median available. Choose whichever one you need. Next hit 2^{nd} and then the 1 button. You should see something like mean(L_1 on the screen. Hit enter, and the calculator will spit out your mean. Easy!

These are just some of the great uses you can put your calculator to. Please feel free to explore online and try out other neat ideas. There are simply too many to list!

TIER ONE'S COMPREHENSIVE MATH CONCEPT REVIEW

As mentioned above, the primary component being tested on this section is math. Therefore, the best way to prepare for it is via intensive study and review of the most common math rules tested on the SAT.

That being said, this is meant to be a resource for review, it is not meant to be a series of workbook exercises. The ideal ways to use this review would be:

1) Skim through and highlight or mark each of the concepts that you know you generally don't remember or understand.

2) Use it as a reference after you identify patterns of problems when using your codification sheets.

IMAGINARY NUMBERS

Under most circumstances, we simply can't take the square root of a negative number. There simply isn't "real" solution, but we can imagine one. To do that, we say:

$$i = \sqrt{(-1)}$$

That mean when you're presented with something like $\sqrt{(-25)}$, you can think of it as $\sqrt{(-1)\times(25)}$ which equals $i\times\sqrt{25}$, or $5i$.

i can be raised to any power, just like a real number. But there is a neat trick to quickly figuring out what the value of *i* to any power is, and it has to do with a cycle.

$i^1 = \sqrt{(-1)}$
$i^2 = \sqrt{(-1)} \times \sqrt{(-1)}$, or just (-1) (in the same way that $\sqrt{2} \times \sqrt{2} = 2$)
$i^3 = i^2 \times i^1 = -1 \times \sqrt{(-1)} = -i$
$i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$
$i^5 = i^4 \times i^1 = 1 \times i = i$ which is back where we started!

That means to find the answer to any *i* with an exponent can be determined by dividing the power by 4 and looking at the remainder.

If the remainder is 1, the solution is the same as the value for i^1, or just i.
If the remainder is 2, the solution is the same as the value for i^2, or just (-1).
If the remainder is 3, the solution is the same as the value for i^3, or just $-i$.
If there is no remainder (4 goes evenly into the power), that's because the power is a multiple of 4, meaning the solution is the same as the value for i^4, or just 1.

But what happens when you have a mixture of real and imaginary numbers like you might see in the solution to a quadratic equation that results in a negative term under the square root?

There is a particular order in which you must write down this "complex number." The standard form is as follows, where a is a and b are constants and i is the same $\sqrt{(-1)}$ we've been talking about:

$$a \pm bi$$

If one complex number is a + bi, a – bi is what we call the conjugate. To get the conjugate, all you have to do is change the sign on the bi term.

Finally, in the same way that we don't like square roots in the denominator, we also don't like to see i there either. To do remove the term, we multiply the numerator and denominator by the denominator's conjugate. For example:

$$\frac{5 + 3i}{3 - 2i} \; x \; \frac{3 + 2i}{3 + 2i}$$

We can do this without changing any values because $(3+2i)/(3+2i)=1$. Multiplying any term by 1 results in no change in value. We can multiply these terms through the FOIL (**F**irst **O**utside **I**nside **L**ast) technique we learned when multiplying polynomials. Just remember that $i^2=(-1)$, and to rewrite your answer in the standard form for a complex number.

Practice Questions:

1) i^{74} = y. What is the value of y?

    a) i
    b) $-i$
    c) 1
    d) -1

2) $(3 + 2i) \times (1 - 4i)$ = z. What is the value of z?

    a) $3 - 8i$
    b) $8 - 3i$
    c) $11 + 10i$
    d) $11 - 10i$

INTEGERS

An integer is any positive or negative whole number, including zero. Examples include:

..., -3, -2, -1, 0, 1, 2, 3, …

That means we must exclude fractions (except when the fraction reduces down to a whole number like 6/2, which equals 3), decimals (both repeating and non-repeating), and complex numbers (see Imaginary Numbers section).

Zero, while an integer, is neither a positive nor a negative integer. So be extra careful to check whether the question specifies positive and/or negative integer values.

Whenever a problem simply says "an integer," you can bet they're actually asking you to use x as a variable replacement for that number.

Practice Questions:

1) Which of the following is NOT and integer:

a) -4/2
b) 0
c) 3
d) 18/5

2) How many positive integers are factors of -14?

a) 0
b) 2
c) 4
d) 6

ORDER OF OPERATIONS (PEMDAS)

Priorities are just as important in math as they are in life. In math, we call our priority rankings an Order of Operations. An operation is a sort of mini-math problem. Multiplying two numbers together or subtracting two terms each count as an operation.

The acronym that almost everyone has to learn at some point that simplifies that Order is PEMDAS:

P: Parentheses (You must complete whatever operations are inside the parentheses before moving on)
E: Exponent (Make sure to deal with any powers on all terms)
M: Multiplication
D: Division (Technically, multiplication and division have the same priority, and you ought to complete each operation as it comes up moving from left to right across the problem.)
A: Addition
S: Subtraction (Just like with multiplication and division, addition and subtraction have the same priority, and you ought to complete each operation as it comes up moving from left to right across the problem.)

If you ever have trouble remembering the order, just recall the phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally!"

Practice Questions:

1) $2 - (3 \times 2^3 - 12)^2 \div 2 = ?$

a) -71
b) -70
c) 70
d) 71

2) For which of the following values of x is $(-3)^x = -3^x$?

I) -3
II) -2
III) -1

a) I only
b) II only
c) I and II.
d) I and III.

ABSOLUTE VALUE

The absolute value of a number is defined as the magnitude (meaning we don't care about the direction, left or right) of a number's distance from 0 on the number line. For example, -2 is 2 units away from 0, and so is +2 for that matter.

That's why when I take the absolute value of a negative number, it turns out to be the same value as the positive version of that number.

$$|-2| = 2 \text{ and } |2| = 2$$

That means when I use x's in an absolute value equation, I need to ensure that I can find both the positive and negative solutions, since both will yield the same result.

$$|x + 2| = 5 \text{ becomes}$$

$x + 2 = 5$ and $x + 2 = -5$

x will then equal both 3 and -7. We can check those answers by plugging them back into the absolute value to confirm.

The same is true for inequalities, although you must be aware that in the inequality that you make negative you must also change the type of inequality symbol you use, just as you would if you were dividing or multiplying that side by a negative number. For instance:

$$|x + 2| \geq 5 \text{ becomes}$$

$x + 2 \geq 5$ and $x + 2 \leq -5$

***Practice Questions*:**

1) $|x| \geq 0$ for which of the following values of x?

   I. All negative numbers
   II. 0
   III. All positive numbers

   a) I only
   b) III only
   c) I and II.
   d) I, II, and III.

2) $|x - 6| + 3 = 4$. What is/are the value(s) of x?

   a) 5
   b) 5 and 7
   c) 7
   d) 7 and 13

CONSECUTIVE INTEGERS

Consecutive integers are just integers that follow in the correct counting sequence. For example, 2, 3, 4, and 5 are consecutive integers.

But how do we represent consecutive integers when you don't know what the starting number is? Or said another way, what do you do when you have to represent the first term as x?

Let's continue to use 2, 3, 4, and 5 as an example. To get from 2 to 3, we have to add 1. To get from 2 to 4, we have to add 2, and to get from 2 to 5, we have to add three. So if we represented 2 as x, we could then say 3 was like x+1, 4 was like x+2, and 5 was like x+3.

The same is true for any consecutive integers. We write them as x, x+1, x+2, x+3, etc. for as many terms as we need.

The method is very similar when I need to deal with consecutive even or odd integers.

Imagine we have 4 consecutive even integers, 2, 4, 6, and 8. To get from 2 to 4, we must add 2. To get from 2 to 6, we must add 4, and to get from 2 to 8, we must add 6. You probably see where this is going. That means if we treated 2 as x, 4 would be x+2, 6 would be x+4, and 8 would be x+6.

The same is true for any consecutive even or odd integers (odd integers have the same differences between their numbers; think about 3, 5, 7, and 9). We write them as x, x+2, x+4, x+6, etc. for as many terms as we need.

***Practice Questions*:**

1) The sum of 3 consecutive integers is 21. What is the largest of these integers?

a) 6
b) 7
c) 8
d) 9

2) The product of 2 positive consecutive odd integers is 195. What is the largest of these integers?

a) 11
b) 13
c) 15
d) 17

FACTORS AND MULTIPLES

The factors of a number are pairs of numbers that multiply to equal the original term. For instance, 1 and 8 multiple to 8, but so do 2 and 4, which means that the number 8 has four factors (1, 2, 4, and 8).

Numbers can have common factors. For instance both 6 and 9 have 3 as factors. The Greatest Common Factor or GCF is the largest shared factor that a set of numbers each possesses.

The multiple of a term is any number that can be evenly divided (the result being a positive integer greater than 1) by that term. For instance, multiples of 12 include 24, 36, 48, 60, 72, etc.

The Least Common Multiple or LCM is the smallest number that is a shared multiple of a set of values. For instance, amongst 3, 5, and 10 the LCM is 30.

Practice Questions:

1) How many distinct factors does q possess if s×t×u=q and s, t, and u are all prime numbers?

a) 3
b) 5
c) 6
d) 8

2) What is the Greatest Common Factor of 32, 8, 128, and 96?

a) 2
b) 4
c) 6
d) 8

FRACTIONS

Fractions are composed of a numerator (the number on top) and denominator. For example, in the fraction, 3/7, 3 is the numerator and 7 is the denominator. You can view the numerator as the number of parts you have and the denominator as how many parts it takes to make up a whole (or a value of 1).

Improper fractions are fractions that have a numerator which is greater than the denominator, meaning that the total value is greater than 1. For instance, 17/5 is an improper fraction. An improper fraction can be converted into a mixed number, or a number that pairs a whole number with a fraction. For example, the 17/5 we just used as an illustration of an improper fraction can be converted into 3 2/5 by simply dividing 17 by 5 and turning the remainder into a fraction. The same method can be applied in turning any improper fraction into a mixed number.

Fractions can sometimes be reduced. In math, while it sometimes doesn't seem like it, our ultimate goal is to make things as simple as possible. If the numerator and denominator of a fraction are found to have a common factor, we can divide each by that factor to simplify. Let's consider 24/36. The GCF of 24 and 36 is 12. 12 goes into 24 twice and into 36 three times. That means we can rewrite the fraction as 2/3. In other words:

$$\frac{24}{36} = \frac{2}{3} \times \frac{12}{12} = \frac{2}{3} \times \frac{1}{1} = \frac{2}{3}$$

Reducing fractions is always a good idea. Working with smaller numbers make problems easier to manipulate. And remember, making things simpler is the goal.

Fractions can also interact with each other in the same way any other number can, meaning they can be added, subtracted, multiplied, and divided from or with one another.

Addition and subtraction of fractions both require finding the common denominator, which can be done by figuring out the LCM of both the denominators in question. Each numerator will then be multiplied by the number necessary to change its respective denominator into the LCM. Finally, we can simply add or subtract the numerators as the problem dictates.

For example, if I wanted to subtract 1/3 from ½, I would first determine the lowest common denominator, which is the LCM of 2 and 3, or 6. Knowing that I have to turn the denominator of 1/3 into 6, I must multiply the numerator and denominator by 2, which doesn't affect the value of the number because I am performing the same operation on both. That results in 2/6. To turn convert ½ into sixths, I would need to multiply both the denominator and numerator by 3, resulting in 3/6. Now, I can finally subtract 2/6 from 3/6, or

$$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

Multiplying fractions requires finding common factors between the denominator of one fraction and the numerator of the other to eliminate, and vice versa. Then we just multiply the two numerators and the two denominators to get our result.

For example, let's say I wanted to multiply $\frac{5}{21} \times \frac{14}{25}$. I notice that 5 can go into

the numerator on the left once and the denominator on the right 5 times. That means I can rewrite the problem as $\frac{1}{21} \times \frac{14}{5}$. I also notice that 7 goes into the

denominator on the left three times and the numerator on the right twice. I can now write the problem in its final form $\frac{1}{3} \times \frac{2}{5}$. This process is known as

cross-cancelling. At this point I can just multiply the two numerators and two denominators to get 2/15, which is the final answer.

We can use the skills we learned in multiplying fractions to divide. The key is to take the reciprocal (or flipping the numerator and the denominator) of the divisor (what's doing the dividing) before multiplying the two terms. For instance, $\frac{5}{21} \div \frac{25}{14}$ is the same as saying $\frac{5}{21} \times \frac{14}{25}$, which we just saw how to do a moment ago.

Practice Questions:

1) If $\frac{24}{5}z = \frac{8}{15}$, find the value of z.

2) $\frac{4}{5} - \frac{3}{8} = y$. What is the value of y?

DECIMALS

Decimals are just another way of writing fractions. We can determine the fraction by understanding how to name decimals. For example, 0.1 is one tenth or 1/10 and 0.43 is spoken as forty-three hundredths (43/100). The denominator of the fraction increases by a factor of 10 (add a 0 to the end of the denominator) as we move each place away from the decimal point itself.

Understanding how to add/subtract and divide/multiply decimals is key for the non-calculator portion of the exam.

We add and subtract decimals by lining them up, such that the decimal points are in the same column. At that point just add or subtract as normal.

$$\begin{array}{r} 0.6325 \\ \underline{-\ 0.423} \\ 0.2095 \end{array}$$

To multiply we line the decimal points up in exactly the same way. We then multiply the two numbers, first ignoring the decimal points. We then sum the number of numbers after the decimal point in both of the original terms. That tells us how many numbers should be after the decimal point in our product. For example, in 0.325 x 0.25, we find that without consideration of the decimals, the answer would be 8125. We then count up the numbers after the decimal point in our original two terms. It turns out there are five. That means we move the decimal point five places to the left in our product, resulting in an answer of 0.08125 (the decimal had to move over one more place than there were numbers in our answer, resulting in the 0 before the 8125).

To divide, we use long division. If we are dividing a decimal number by a decimal, we move the point over to the right on both terms the number of places that would make the divisior into an integer. For instance, if 0.35 were to be divided by 0.2, we would move the decimal over to the right by one place on both terms, changing the problem to 3.5/2. Using long division we find the answer to be 1.75.

Practice Questions:

Answer without the use of a calculator:

1) 2.6723 – 0.9526

2) 6.67/0.47

PROPORTIONS, RATIOS AND PARTS OF A WHOLE

Proportions are great ways of relating quantities that have similar relationships. For instance, imagine the key of a map tells me that every 3 in. on the map equates to 5 mi. in reality. If I know that relationship, I can use proportions to find out how many miles 17 in. on the map represents. What I can do is represent the first relationship (the one mentioned in the map key) as a fraction, and set it equal to the second relationship (the one where I only know the inches on the map) in which I replace the unknown quantity with an "y."

$$\frac{3 \text{ in.}}{5 \text{ mi.}} = \frac{17 \text{ in.}}{y \text{ mi.}}$$

Now we can cross multiply (meaning multiply the numerator of the proportion on the left with the denominator of the proportion on the right and the denominator of the proportion on the left with the numerator of the proportion on the right) to get 3y=17x5, or 3y=85.

Now we just divide both sides by 3 to get y=28 and 2/3 mi.

As a side note, let's talk about the ways in which proportions can be written. Using the map example above, the inches to miles relationship could have been written as either "3 to 5" or "3:5." We sometimes call proportions written in this way ratios.

Sometimes the ACT likes to represent ratios as compared to the number 1. To do so, you simply divide both of the numbers by the second term in the relationship. So in the inches to miles relationship above, the ratios could be written as "3/5:1" or "3/5 to 1."

Ratios should also be used when comparing parts to wholes. For instance, let's say I know that a sector of a circle has an arc measure of 4π and a central angle equal to 60°. I should be able to use the relationship of the degree measure of the central angle and the fact that a circle has a total degree measure of 360 to determine the circumference of the whole circle through the following proportion:

$$\frac{60}{360} = \frac{4\pi}{C}$$

We can simplify 60/360 to 1/6, and then cross-multiply to get C=24π.

Ratios can also be used when discussing the relationship of parts without reference to the whole. For instance, if I say that a particular fruit juice is made by mixing 3 parts apple juice to 2 parts orange juice to 1 part pineapple juice, we can say there is a ratio of 3:2:1. Notice we haven't said anything about the whole quantity (which would be made up of 6 parts: 3+2+1) yet. If a problem asked us to solve for the number of cups of orange juice needed to make 24 cups of the mixture, we could set up the following proportion:

$\frac{3}{6} = \frac{x}{24}$ since 3 is the number of parts of OJ, 6 is the total number parts in any given quantity of mixture, and 24 is the total number of cups in the specific mixture we're making.

x, then, equals 12 cups!

Practice Questions:

1) A cartoonist draws people in his Sunday morning comic strip at $1/20^{th}$ scale. What would be the actual height, in feet, of a dog drawn to be 1.5 inches tall?

2) A pizza is cut in to twelve slices for a math class end of semester party. One student decides to measure the curved length of his crust with a string, and finds it to be 0.5 in in length. What is the area of the whole pizza?

PERCENTS

In Latin, percent literally means "out of 100," meaning I can actually rewrite anything given to me as a percent as a fraction with 100 in the denominator. For instance, 92% can be rewritten as 92/100. If I so desired, I could then simplify that fraction down to 23/25.

This definition of percent also explains why I can rewrite a percent as decimal by simply moving the decimal of the percent two digits to the left. As a result, 92% can be rewritten as 0.92.

Since the SAT has a no calculator portion it is useful to have the following fractions/decimals/percents memorized:

1/8=0.125=12.5%
1/5=0.2=20%
1/4=0.25=25%
1/3=0.3333=33.33%
3/8=0.375=37.5%
2/5=0.4=40%
1/2=0.5=50%
3/5=0.6=60%
5/8=0.625=62.5%
2/3=0.6666=66.66%
3/4=0.75=75%
4/5=0.8=80%
7/8=87.5%
8/8=5/5=4/4=3/3=2/2=1=100%

Both the fraction and decimal versions of a percent are real numbers and can be used to equations. I must change a percent to a real number to use it in any further math operations.

We can determine what percent one number is of another number by two methods. This can be illustrated with variables. Imagine I asked "12 is what percent of 96?" We can either find the decimal value of a÷b and move the decimal point to places to the right, or we can create a proportion.

$$\frac{Y}{100} = \frac{12}{96}$$

We can then cross-multiply to get 96Y=1200. Once we divide both sides by 96, we get Y=12.5. Therefore, we can say that 12 is 12.5% of 96.

When a number changes (like the amount of money you have in the bank), we can compare the new number to the original to describe percent change. To find the percent of this increase or decrease, we simply divide the difference from the original number (found by subtracting the original from the new) by the original number. For example, let's say my bank balance increased from $5,000 to $7,000. We would find the percent change as follows:

$$\frac{7{,}000 - 5{,}000}{5{,}000} = 0.40 \text{ or } 40\%$$

Lastly, let's consider an application of percents. Frequently, we want to say that sizes, costs, weights, etc. can change by a certain percentage amount. If something is to increase by Y %, we can multiply the original number by (1+Y/100). We are simply adding to 1 (which can be understood to mean 100%, or the original amount) and the decimal or fraction version of the percentage increase. Likewise, if I decrease a quantity by Z %, I just multiply the original amount by (1-Z/100).

Practice Questions:

1) A shop normally sells a hat for 40% more than the wholesale cost. Because of this high markup, the shop is now going out of business. In order to liquidate its assets, the owner has marked all goods as 15% off wholesale cost. If you were to buy a hat for $10 at this sale, what would the hat have cost you prior to the sale?

2) You receive a score of 86% on a history exam. If the exam was scored out of 300 points, how many points did you receive?

PROBABILITY

Probability is defined as the likelihood of an event occurring given a certain number of possible outcomes.

The most basic way to understand the concept is through applying probability to a coin flip. There are two possible outcomes: either the coin lands on heads or tails. If I wanted to find the probability of the coin landing heads up, that is one of the possible outcomes. In total then I would say that the probability is 1/2 , or the number potential desired outcomes (or "successes") divided by the total number of all possible outcomes.

$$\text{Probability} = \frac{\text{\# of "successes" or desired outcomes}}{\text{\# of all possible outcomes}}$$

To find the probability of successive events (one event after the other), we simply multiply each of the probabilities together. The product is the probability. Returning to the coin example, the probability of flipping a coin and having it land tails-side up 3 times in a row would be (1/2)x(1/2)x(1/2), or 1/8.

Probability can be expressed as a fraction, decimal, or percentage.

Practice Questions:

1) A bag of marbles has 10 red marbles, 6 green marbles, and 4 blue marbles. What is the probability that I will pull out 2 red marbles in a row, if I put the marble from my first draw after checking the color?

2) A board made of 3 concentric circles is used in a darts game. The centermost (and smallest circle) is worth 10 points and has a radius of 3 in. The middle circle has a radius of 5 cm and is worth 5 points. The outermost circle has a radius of 7 in. and is worth 1 point. What is the probability of receiving 10 points if I randomly throw a dart at the target?

RATE

Every task takes a certain amount of time to complete, whether it is driving to the grocery store, manufacturing a product, or completing an ACT exam. When we divide the units of the task (miles driven, products manufactured, or questions answered) by the time it takes (in seconds, minutes, hours, days, or years), we arrive at the rate at which the task was accomplished.

One of the most basic formulae involving a rate that you may know is d=rt, or distance=rate x time. We tend to use this for objects or people that are travelling at particular speeds for specific distances during a time frame.

Another place you may encounter the word rate is in problems pertaining to investments. This usage is different than what we just described above and has its own unique rules. The standard formula for what we call compound interest is

$$A=P(1+r/n)^{nt}$$

where A is the amount of money you end up with, P is the principal (original) amount of money invested, r is the rate (or percentage converted to a decimal) at which the money is invested, n is the number of times per year the principal is compounded (how many times does the bank calculate interest gained per year), and t is the number of years the money is left in the account.

The last place the ACT may use the word rate is in reference to slope, which is a rate at which the y values are changing relative to the x values (hence the formula for slope, $m=\Delta y/\Delta x$). See section on slope for more details.

Practice Questions:

1) If you want to invest $8000 in a certificate of deposit (CD) for 5 years, which would be the smarter business decision, placing all of your money in a CD at a rate of 4% that compounds tri-annually or one at a rate of 6% that compounds annually?

2) If Machine A makes 500 widgets in 3 hours and Machine B makes 200 widgets in 5 hours, how many widgets will be made if both machines are working constantly for 30 hours?

MEAN, MEDIAN, MODE, & RANGE

The mean, or average, of a set of terms is the quotient of the sum of all of the terms divided by the total number of terms, or

$$\text{Average/Mean} = \frac{\text{sum of \#'s}}{\text{Total number of \#'s}}$$

Too often students forget that this formula is just about finding the average. We can use it to find the total number of terms and the sum of the terms as well.

The mode is the most frequently occurring term in a set of terms. For instance, the mode of the list

3, 3, 3, 4, 4, 5, 5, 6, 6, 6, 6, 7

is 6 because it appears four times, more than any other term. A list can have more than one mode if two terms appear an equal number of times.

The range is simply the difference between the highest and lowest value terms in the list. In the previous list the range would be 4 (found by subtracting 7-3).

The median is a unique quantity; it can be viewed as the number that divides the greater half of the terms from the lesser half. For example in the following list

5, 8, 21, 75, 107

21 is the median terms because it separates the smallest two numbers (5 and 8) from the greatest two values (75 and 107). One of the methods to determine the median is to make sure the list of terms is written in ascending value and then find the middle term. That will be your median if there are an odd number of terms. What happens when we have an even number of terms, though? For example, what if we had the following list?

5, 8, 15, 21, 75, 107

With an even number of terms we take the <u>two</u> "middle" values and find their average (or simply divide their sum by two). In this case 15 and 21 are the two "middle" values. Their sum is 36. When I divide that by two, I get 18, which is the median of that list of terms.

Another method for finding the median also depends on whether the number of terms is even or odd. If there is an odd number of terms, we divide that number of terms by 2 and round up. The resulting number will tell us the term number where we can find the median. For example, if I had a list of 37 terms, I would divide 37 by 2 to get 18.5. When I round 18.5 up, I end up with 19. That tells me that the 19^{th} term is the median.

The process starts the same if I have an even number of terms. First divide the number of terms by 2. Now instead of rounding up, we find that term and the next highest term so that we can divide their sum by two. For example, if I had a list of 36 terms, I would first divide 36 by 2 to get 18. That tells me I need to look for the 18^{th} and 19^{th} terms on my list, add those two numbers together, and divide their sum by two to get my median.

This method is particularly useful for tables of values and lists with many terms.

Do not worry if this method doesn't make sense! You can always write out the terms and use the first method we talked about.

***Practice Questions*:**

1) If I stacked 8 tables, one atop the other, with an average height of 40", would a 6' person who can only raise his hands to the level of his head be able to screw in a light bulb in a socket recessed 2" into a ceiling 30' above the floor?

2) 25 people are given a survey at the mall asking the year their cars were manufactured. The results are given below.

Year of Manufacture	**Number of Respondents**
2015	4
2014	5
2013	3
2012	0
2011	1
2010	4
2009	0
2008	0
2007	3
2006	0
2005	2
2004	3

If 3 more people take the survey and each has a car manufactured in 2008, what will be the mean, median, and mode of the survey?

SLOPE & MIDPOINT

The slope of a line can be determined when the x and y coordinates of any two points on that line are known. We call these points (x_1, y_1) and (x_2, y_2). From these points we can find the slope, or m, through the following formula:

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{\Delta y}{\Delta x}$$

where Δ (pronounced delta) just means "change in." The x and y values of the two points are different. How different they are, or how much they've changed, is what we find when we subtract one from the other. Also note how the change in y is in the numerator and the change in x is in the denominator. We sometimes call this "rise over run" since the movement in the y direction is vertical (a rise) and in the x direction is horizontal (a run).

While we're on the subject of slope, it is useful to note that two lines that are parallel have the same slope, which is why they never touch. Two lines that intersect at a 90° angle are called perpendicular, and their slopes are negative reciprocals of one another.

The coordinates of the midpoint (or the point that divides a segment into two smaller, congruent segments) can be found using the following formula:

$$([x_1+x_2]/2\ ,\ [y_1+y_2]/2)$$

***Practice Questions*:**

1) The peak of a 10,000 ft. mountain is reached by traversing a horizontal distance of 15,000 ft. Assuming a smooth slope, what is the slope of the climb up the mountainside from the base to the peak?

2) 4 lines intersect on a plane such that the shape bounded by the four lines is a square. If the slope of one of the lines is 2, what is the slope of another of the lines with which it intersects?

LINEAR EQUATIONS & THEIR FUNCTIONS

You've probably been working with linear equations since middle school, but a quick refresher never hurts.

First, you know that no variable in a fully expanded (meaning that everything has been multiplied, FOIL'ed, etc.) linear equation (two expressions separated by an equals sign) can have a power greater than one. The variable in any equation is standing in place of a number. We're not sure which number until we solve the equation, but as long as we remember that the rules that would apply to a normal constant also apply to the variable, we should be fine.

Second, the Golden Rule of Equations states: "Do unto one side as you do unto the other." If we subtract 2 from one side of the equals sign, we need to subtract 2 from the other side of the equals sign. If we multiply by 4/5 on one side, we multiply by 4/5 on the other side. If we raise one side to the 3rd power, we raise the other side to the 3rd power. I think you get the point.

To solve a linear equation, we first combine any like terms (i.e., make sure that if I see 5x-2x, I change that to 3x) Then we isolate whichever variable we're looking for by moving (through the Golden Rule) everything else to the other side of the equals sign.

For example, lets say I wanted to solve for z in the following equation:

$$\frac{4x+z}{8}=2$$

To solve, I somehow need to separate the 8 and the 4x from the z term on the one side. To do so, I follow a reverse order of operations (remember PEMDAS). Since z (and the 4x for that matter) is being divided by 8, I will deal with that first. To move that 8 over to the other side, I have to perform the opposite operation from what it is currently doing. Since 8 is dividing right now, I must multiply both sides by that 8 instead.

$$8 \bullet \frac{4x+z}{8}=2 \bullet 8$$

The 8 in the denominator of the term on the left hand side will cancel out because of the 8 that we multiplied it by, and the 2 on the right hand side will become 16 since it was also multiplied by 8. We now have

$$4x+z=16$$

To get the z by itself, all we have to do now is subtract 4x from both sides. In so doing, we end up with no x's on the left hand side and 16-4x on the right hand side. As a result

$$z=16-4x$$

As a quick side-note, whenever a variable is multiplied by a fraction like $\frac{3x}{5}$ the quickest way to isolate the variable (let's say we were looking to find x) is to multiply both sides by the reciprocal of that fraction, in this case 5/3. Sure, you could multiply both sides by 5 first and then divide both sides by 3, but why do two steps when one gets the job done equally well?

Sometimes a question will ask for a variable in terms of other variables. The question will often be phrased as "Find x in terms of y and z" or "What is x in terms of y and z?" Of course, we could be using other variables, but whatever comes after the word "find" or the variable after "what is" is what you are trying to isolate in the same way we isolated z in the previous few paragraphs.

Linear functions are equations of two variables, usually x and y or x and f(x). x is usually the independent variable, or the variable we would plug in first to find y, which is, as a result, called the dependent variable (it depends on x for its value). f(x) is another, more powerful way to say y. It's more useful because it tells us, "this is the y value when x equals a particular value." That's a lot more information than just a plain y. We can even make composite functions in which we insert one function for x in a different function like so:

If $f(x)=2x+1$ and $g(x)=3x-1$, then $f(g(x))$ would be
$$2(3x-1)+1=6x-2+1=6x-1$$

A point on a line is written as (*a*,*b*), where a is some coordinate along the x or horizontal axis and b is some coordinate along the y or vertical axis. Substituting *a* in for the independent variable will always yield *b*, the dependent variable.

Linear functions have a domain (the possible x-values for a function) from negative infinity to positive infinity and a range (the possible y-values for a function) of negative infinity to positive infinity. Another way to notate this is a domain and range of $(-\infty,\infty)$. The parentheses mean that the values are exclusive (because infinity is so large we can't bound it). If the range had been from 2 inclusive (meaning 2 is on the graph) to 5 exclusive (meaning the function never reaches a y-value of 5), we would write the range as [2,5).

The most common form for a linear function is what we call the slope-intercept form, which is written as

$$y=mx+b$$

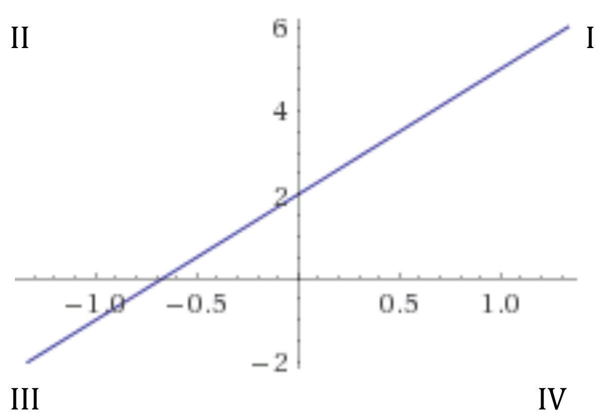
where y is the dependent variable, m is the slope, x is the independent variable, and b is the y coordinate of the y-intercept. A y-intercept is the point at which the function crosses the y-axis. It is almost always easiest to re-write a function into the slope-intercept form, although you also need to know the standard form for the SAT:

$$Ax+By=C$$

where A, B, and C are all integers. In this form, the slope is –A/B, the y-intercept is C/B, and the x-intercept is C/A.

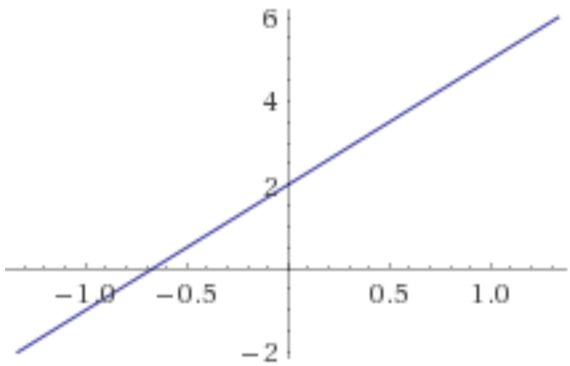
Slope-intercept form also makes it very easy to quickly sketch the graph of a given linear function. We can easily spot at least one point on the line: the y-intercept, which should just be (0,b). From there, we can use the slope to guide us. First, remember that Δy is the numerator of the slope and Δx is the denominator. To find another point on the line with a positive slope, we will go up Δy units and to the right Δx or down Δy units and to the left Δx units (which makes sense because a negative divided by a negative is still a positive). With a negative slope, we can go up Δy units and to the left Δx units or down Δy units and to the right Δx units.

As a side note, it is sometimes useful to know the numbers for the various quadrants (the four areas bounded by two of the axes in a standard xy coordinate system). In the following graph, please note the Roman numerals indicating the number of the quadrant



***Practice Questions*:**

1) What is the equation of the following line?



2) Line *l* passes through the origin and is perpendicular to line *z*. If line z can be modeled by the function y=2x+k and the two lines intersect at the point (c, c+4), what is the value of c?

SYSTEMS OF EQUATIONS

There are two reasons to use systems of equations. The first is to find the point of intersection for two functions (which can also be done by setting each function equal to y and then both functions equal to each other. Find the x-value that way, and plug in the x-value into either equation to get the y-value for the point).

The other is to find the values for two unknowns. If we can find two different ways to relate those unknowns to each other, we have a system of equations. For instance, let's say you have a handful of change that is only made up of nickels and dimes. You know that the total amount the cashier just handed you is $2.05, and you also saw that there were 21 coins in the group. Those are two different

relationships, one for value and one for quantity. So long as we keep it to two variables, we can figure out how many each of dimes and nickels you have.
First, let's call the number of dimes "d" and the number of nickels "n." We know that when we add up the two quantities (or the number of each), we should end up with 21 coins. The equation expressing that is $d+n=21$.

We also know that a dime is worth \$0.10 and a nickel is worth \$0.05. That means if you wanted to figure out how much money you could multiply the value of the coin by the number of coins you have. When we combine the amount of money you have in nickels with the amount of money you have in dimes, it should add up to \$2.05, meaning we can come up with an equation saying $0.10d + 0.05n = 2.05$.

We can now line up our two equations as a system:

$$\begin{aligned} d + \quad n &= 21 \\ 0.10d + 0.05n &= 2.05 \end{aligned}$$

We can solve this system through one of three ways.

The first is graphing, which is really impractical unless you're great with a graphing calculator. If you are, simply find the equations in terms of a single variable (e.g., $d=(21-n)$ and $d=(2.05-0.05n)/0.10$), graph them on the calculator, and use the appropriate keystrokes to find the point of intersection. The x and y coordinates of the point will give you the values of your variables with the y corresponding to the variable you found the both equations in terms of. If I use the two equations in this paragraph, I find the point of intersection at (1,20), telling me there are 20 dimes and 1 nickel in the pile.

The second is substitution. This method is fairly straightforward. First, isolate one of the two variables in one of the equations. In the system above, if we isolated d in the first equation we would wind up with $d=21-n$. Then we substitute the value we found for that isolated variable for the same variable in the other equation. In our case, we isolated d in the first equation, so we're going to plug in $21-n$ in for d in the second equation, yielding $0.10(21-n)+0.05n=2.05$. When we distribute the 0.10, we get $2.1-0.10n+0.05n=2.05$. When we combine the two n terms and subtract 2.1 from both sides, we get $-0.05n=-0.05$. When we divide both sides by -0.05, we see $n=1$. Now that we know the value for n, we can substitute that number value in for n in the equation we found in the first place ($d=21-n$), which tells us $d=21-(1)$, or $d=20$.

The third method is elimination. The key to the elimination method is realizing that if we line up the two equations, we can actually add them up. For example

$$\begin{array}{rr} & d + \quad n = 21 \\ + & 0.10d+0.05n=2.05 \\ \hline & 1.10d+1.05n=23.05 \end{array}$$

Consequently, we if we alter one of the equations such that the coefficient on one of the variables in one equation is the negative of its corresponding variable in the other equation, when we add them up, that variable will cancel out. We can do this sort of rearranging by multiplying each of the terms in one of the equations by a constant to produce the desired result. For example, let's say I wanted to solve for d first in the system we've been using. That means I need to eliminate the n. To do so, I will multiply each term in the first equation by -0.05 (doing so to each term doesn't change the overall value of the equation; remember the Golden Rule), so that when I add the two equations up the now $-0.05n$ in the first equation will cancel out the $+0.05n$ in the second equation like so:

$$
\begin{array}{rl}
 & -0.05d-0.05n=-1.05 \\
+ & \underline{0.10d+0.05n=2.05} \\
 & 0.05d \qquad\qquad =1
\end{array}
$$

I now have a simple equation of 0.05d=1, which can be solved by dividing both sides by 0.05, yielding d=20. Since d=20, I can plug that value for d into my original first equation (the d+n=21) to find (20)+n=21. When I subtract 20 from each side, I find n=1.

Notice how the answer is the same in each of these three methods, 20 dimes and 1 nickel.

It is important to note that each of these methods is also applicable to systems of inequalities. You just need to make sure that the inequality symbol is the same for both members of the system.

You may also find that you wind up with one of two unique end points. If your system ultimately simplifies to a number or set of terms equaling the same number or set of terms, then the system has infinitely many solutions. This can be easy to spot when one equation is simply a multiple of the other. The other possibility is that you end up with one number equaling a different number (e.g., 1=4). We know that cannot be possible, so after double checking our work, we can conclude that there are no solutions to this particular system of equations.

Practice Questions:

1) Tickets to a high school drama production are \$4 for students and \$15 for all others. If there were 200 tickets sold and the box office took in \$1845 in sales, how many of each ticket type were sold?

2) Find the point of intersection of lines *l* and *k*, where *l* is modeled by 4x+3y=25 and *k* by 2x+7y=29.

EXPONENTS AND ROOTS

Exponents tell us how many times a particular term is multiplied by itself to result in a particular product. For example, $2^5=2x2x2x2x2=32$. We frequently read that aloud as "2 to the 5^{th} power" or "2 to the power of 5." The two is considered the base.

Roots do the reverse. When I take the 5^{th} root of 32 (or $^5\surd 32$), I'm essentially asking, "what number times itself 5 times is equal to 32?" We know from the previous paragraph that the answer is 2. Whenever you don't see a number associated with the root symbol, it is assumed to be a square root (or asking for what number times itself is equal to the number inside the square root).

Negative numbers taken to an even power always result in a positive solution, since I'm multiplying a negative number by itself an even number of times. For example, $(-2)^4=(-2)x(-2)x(-2)x(-2)=16$.

That means that with even roots, we have the unique situation of having two answers. In the √4, for instance, it could be that either 2 or -2 is the correct answer (note that 2x2 and (-2)x(-2) both equal 4). As a result, we must say the answer is ±2. Normally, though, we tend to assume the positive, or principal, root.

Similarly, we cannot take even roots of negative numbers, because we can never produce a negative number by multiplying a given value by itself an even number of times; however, we can take odd roots of negative numbers, because a negative number multiplied by itself an odd number of times is still negative. For example, $^3\surd(-27)$ is (-3) because (-3)x(-3)x(-3)=(-27).

When I see a fraction exponent, the numerator tells me what power I'm raising the base to, and the denominator tells me what sort of root I'm applying to the number. I can perform those operations in whatever order I like. For example, $8^{2/3}$ can be understood as $^3\surd(8^2)={}^3\surd 64$, or as $({}^3\surd 8)^2=(2)^2$. Both result in an answer of 4. This also explains why one way to view roots is simply to take the number to a fraction power with a numerator of one. For example √4 can be rewritten as $(4)^{1/2}$.

Sometimes you will see a negative exponent. The negative sign first tells you to take the reciprocal of whatever is being raised to that power. Then you take that reciprocal to the positive version of whatever the exponent is. For instance, $(1/5)^{-2}$ requires us to first take the reciprocal of (1/5), which is 5/1 or just 5. We then take that 5 to the positive 2 power, resulting in 25 as our solution.

When taking a fraction to a power, like $(2/3)^3$, you simply apply that power to both the numerator and denominator separately. In this case it would result in $2^3/3^3$, or 8/27. The same is true of roots. √(25/49) becomes √25/√49, or 5/7.

One problem is that we don't really want to have a root in the denominator. The way we rectify that problem is to multiply the numerator and denominator by that very root itself. For example, let's say I want have $(4/7)^{1/2}$. We can rewrite that as (√4)/(√7), which simplifies to 2/√7. To get rid of that square root in the denominator we can multiply both the numerator and denominator by √7 as shown below:

$$\frac{2}{\sqrt{7}} \text{ x } \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

Roots can also be simplified. It is a property of square roots that if I can separate terms as a product under the square root sign, I can view them as individual square roots. Let's take $\sqrt{50}$ as an example. It could be rewritten as $\sqrt{(25x2)}$, which itself can be rewritten as $\sqrt{25}$ x $\sqrt{2}$. We know that $\sqrt{25}=5$, so the whole product can be rewritten as $5\sqrt{2}$. You will often see answers in a simplified form.

That being said, with both exponents and roots, while terms taken to a power or rooted that are being multiplied or divided may have the same operation done to each term, the same is not true for terms that are being added or subtracted. For example, $(2x)^2=4x^2$. The 2 and the x are multiplying within the parentheses, so I can square each term. Likewise, $\sqrt{(x/4)}$ can be rewritten as $\pm(\sqrt{x})/2$ because I can take the square root of both x and 4 since they form a quotient.

On the other hand $(x+2)^2 \neq x^2+4$. I cannot simply square both the x and the 2. To prove that, imagine the x=3; in that case the expression would be $(3+2)^2$. According to Order of Operations, I must first add the 3 and the 2 within the parentheses to get 5 before I square that 5 and get 25. If $(x+2)^2=x^2+4$, then I should be able to plug 3 in for x on either side and get the same result. We've already discussed the left hand side of this equation, but what about the right hand. That would be $(3)^2+4$, or 9+4=13. $13 \neq 25$, so we know that $(x+2)^2 \neq x^2+4$.

The same is true for square roots. I cannot say that $\sqrt{(x^2+25)}=x+5$. I can say that $\sqrt{(x+25)^2}=\pm(x+25)$, however, because the whole expression (x+25) is being squared and then square-rooted.

Let's now move on to the Laws of Exponents. If I have a common base, there are several laws that allow me to do some quick manipulations.

1) When two terms with a common base are multiplied, their exponents can be added.
   a. $x^a \bullet x^b = x^{a+b}$
2) When two terms with a common base are divided, their exponents can be subtracted
   a. $x^a/x^b = x^{a-b}$
3) When a term that is already possesses an exponent is raised to a power, those two powers multiply
   a. $(x^a)^b = x^{a \cdot b}$

Two final thoughts on exponents

First, $-(2)^2 \neq (-2)^2$. Because of the Order of Operations, the term on the left requires me to take the 2 to the second power and then multiply it by a negative, resulting in -4 as an answer. The term on the right is (-2) to the second power, or (-2)x(-2), or 4. As we know, $-4 \neq 4$.

Second, fractions raised to positive, non-fraction powers always become smaller in size, since the denominator will increase.

Practice Questions:

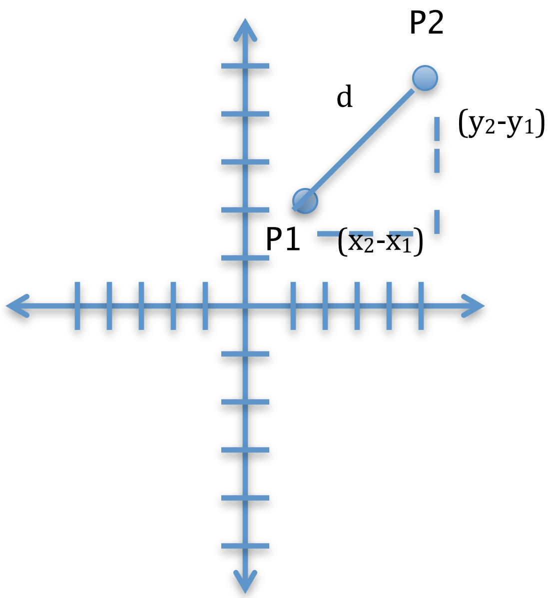
1) If $4 = \sqrt[3]{(x+7)^6}$, solve for any negative values of x?

2) If $2^a \bullet 2^b = 2^{a-b+4}$, what is the value of b?

DISTANCE FORMULA AND PYTHAGOREAN THEOREM

Imagine we had two points, P1 (x_1,y_1) and P2 (x_2,y_2). How would we find the distance between these two points on a plane?

Let's imagine they are both in the first quadrant of a standard Cartesian coordinate system. We'll call the distance between them d. We can draw our own little dashed lines down from P2 and across from P1 to make a triangle. As we can see, the horizontal leg of that triangle will have a length of (x_2-x_1). After all, it's a horizontal line! The y value won't change. Since the only change in that line is a change in the x direction, that's how we calculate its length. Likewise, the vertical line has a length of (y_2-y_1), for the same reason, except this time only the y values are changing.



We now have all of our sides in terms of variables. Because we have a straight horizontal line and a straight vertical line (our dashed lines we drew ourselves), we know they meet at a right angle.

That means this is a right triangle we have now, which tells us we can use the Pythagorean Theorem, or $a^2+b^2=c^2$, where c is the hypotenuse and a and b are the legs of the right triangle. The Pythagorean Theorem can only be used for right triangles and is one of the most versatile formulae on the ACT. In our specific case here,

$$d^2=(x_2-x_1)^2+(y_2-y_1)^2$$

Because we just want to know d, the distance between the two points, we'll have to take the square root of both sides to discover that

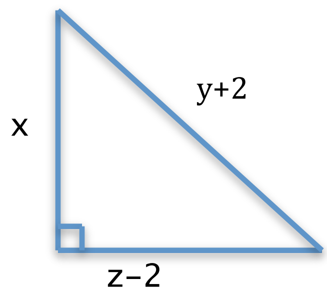
$$d=\sqrt{x_2-x_1)^2+(y_2-y_1)^2}$$

And lo and behold, we have just derived the distance formula.

***Practice Questions*:**

1) The distance between points P (2,4) and Q (z,7) is 4. If z Is a positive number, what is the value of z?

2) Find x in terms of y and z.



MONOMIALS, BINOMIALS & FOIL

A monomial is the product of any non-negative (meaning including 0) power of variables. Examples of monomial include 2, 7x, -52xy, $86x^2z^{74}$, etc.

A binomial is the sum or difference of two monomials, while a trinomial is the sum/difference of three monomials. Really any sum or difference of some number of monomials is called a polynomial.

First, let's learn how to multiply monomials. It's really a matter of multiplying like terms, as it were. Imagine I had $(4x^2y^3z^4)(-2x^3y^2z)$. First, let's multiply the coefficients, 4 and -2, which results in -8. Then we'll multiply the x terms (x^2 and x^3) to get x^5, the y terms (y^3 and y^2) to get y^5, and the z terms (z^4 and z) to get z^5. Since everything is multiplied together, our final product is $-8x^5y^5z^5$. The same is true for dividing monomials. Focus on the common terms.

We tend to focus on binomials for reasons we will discuss later (mainly for their use as zeroes), but let's first begin with multiplication of binomials. Let's say we wanted to find the product of (x+2)(x+3). To do so, we need to multiply each term in one binomial by each term of the other binomial (the same is true for all polynomials) and add those products together. With binomials, we have an acronym for system that ensures we do just that called FOIL, or First Outside Inside Last. Each term in the product has a position as labeled below.

$$\overbrace{(x + 2)\,(x}^{\text{First}} + 3) \qquad \overbrace{(x + \underbrace{2)}_{\text{Inside}}\,(x}^{\text{Outside}} + 3)$$

$$(x + \underbrace{2)\,(x + 3}_{\text{Last}})$$

First, we multiply the First terms, x and x, to get x^2. Then we multiply the Outside terms, x and 3, to get 3x. Next, the Inside terms, 2 and x, to get 2x. And, finally, the Last terms, 2 and 3, to get 6. When we add them all up we get $x^2+3x+2x+6$, and when like terms are combined we finish with x^2+5x+6.

Note that (x+2) and (x+3) multiply to result in x^2+5x+6, hence we call (x+2) and (x+3) factors of x^2+5x+6. In fact, the reverse process to FOIL'ing (i.e., coming up with the factors) is called factoring.

The most frequent polynomials you'll have to factor are quadratic (highest power of two) trinomials with a leading coefficient (the coefficient on the highest power variable) of 1. They will generally look something like ax^2+bx+c, where a, b, and c are constants. The first step is to try and factor out any constants so that we're dealing with a leading coefficient of 1 in the first place. For example, imagine the trinomial was $3x^2+15x+18$. Notice how each of the coefficients is a multiple of 3. That means I can "pull out" or "factor out" a 3 from each term, rewriting the trinomial as $3(x^2+5x+6)$. That way we can then move on to factor the expression inside the parentheses.

Next, we know that the factors have to look something like (x) and (x). What goes in those empty spots is determined by both b and c. The two numbers need to multiply to be c and sum up to b. For instance, recall how the trinomial x^2+5x+6 had factors of (x+2) and (x+3). Note that +3 and +2 (the numbers in those empty slots) multiply to be 6 (the c term) and add up to be 5 (the b term).
It might get trickier with negative terms, and you'll have to try several different combinations of numbers to land on the correct pair; however, I promise it is do-able.

It gets much trickier when the leading coefficient is a number other than 1. Odds are that the ACT will use a prime number instead of one making the new format (ax)(x). Now what goes in the two empty slots is a bit trickier. The b value will be the sum of the number put in the first slot and the product of a and the number put in the second slot, while the c value will be the same as before (the product of the two numbers that fill the slots).

To illustrate, let's consider $2x^2-8x+8$. First, write our factors as (2x)(x).

Then let's look at factors of +8 to put in the two slots. I notice that b is a negative number, so I'm probably going to be dealing with the negative factors of positive 8, either -1 and -8 or -2 and -4. No matter how I place the -1 and -8, I can't wind up with a b=-8, but when I put the -4 in the first slot and the -2 in the second, I can do just that since I multiply the -2 times the a value and add that product to the number we put in the other slot (-4). As a result, our factorization is (2x-4)(x-2).

One special case exists that you should be able to factor almost automatically called the Difference of Squares. The Difference of Squares is exactly what the name implies; we subtract one square from another like $4x^2-25$. Both $4x^2$ and 25 are both squares. In general then, the form looks like d^2-f^2. The factorization of that general form is (d-f)(d+f). So for our $4x^2-25$, the factorization is (2x-5)(2x+5).

***Practice Questions*:**

1) Factor $6x^2-3x-18$.

2) Factor $-16x^2+144$.

3) Simplify $\dfrac{2x^5yz^3}{4x^3y^2z^2}$

QUADRATIC EQUATIONS

We've already seen some quadratic expressions, but not equations. The key to solving a quadratic equation is to make sure one side is set equal to zero. At that point we can then factor or use the quadratic equation to solve.

If we factor, the point to realize here is that we are coming up with x values that would make the whole expression equal to 0. In other words, what are the x values that turn a single factor into 0 (since if one factor is 0, the whole product is 0)? We figure that out by setting each of our factors equal to 0 and solving for x. For example in the quadratic equation $2x^2+3x=-1$, we would first add 1 to both sides to get $2x^2+3x+1=0$. Then, after factoring, we wind up with $(2x+1)(x+1)=0$. Finally, we set each of the factors equal to 0 and solve for x as shown:

$$(2x+1)=0$$

Subtract one on both sides and then divide by 2 to get

$$x=-1/2$$

And $(x+1)=0$

Subtract 1 from both sides to get

$$x=-1$$

That tells us that the solutions, or the zeroes, of the quadratic equation in question are -1/2 and -1.

The other way to solve for those zeroes would be to use The Quadratic Formula, which looks at the form $ax^2+bx+c=0$ and gives us a formula to solve for x.

$$x=\frac{(-b)\pm\sqrt{(b^2-4ac)}}{2a}$$

If the quadratic equation in question isn't factorable or if it just looks too hard, the Quadratic Formula, if used correctly, is guaranteed to give the correct solution. Please note the ± symbol, indicating that you will need to separate the problem into two separate problems, one that uses a + sign there and another that uses a – sign.

We must be careful, though, if we are responding to a word problem using this method. Many times we will find an answer that is extraneous, or possible using the mathematical approach but inconsistent with reality. For instance, if we find a negative and positive value for the time it takes a ball to fall to the earth after being thrown, we know the negative value, which indicates time in the past, must be extraneous (unless you've invented a time machine!).

Practice Questions:

1) Find the solution(s) of $4x^2=16$.

2) Find the zeroes of $x^2+4x+2=0$.

TRANSLATIONS & FUNCTION PROPERTIES

Function translations can cause compression, stretching, vertical shifts, and horizontal shifts. Let's first discuss the topic generally and then specifically look at translations of quadratic functions.

If a function can be written as f(x), let us say that we could also write it as af(x-h)+k, where a dictates a compression or stretch, h corresponds to horizontal shift, and k indicates a vertical shift. a will also cause the function it to be flipped upside down if it is negative. When the absolute value of a is a fraction, the graph is sort of horizontally stretched out (made fatter), but when the absolute value of a is greater than 1, the function is vertically stretched (made thinner). When h is positive the function moves to the right, and when h is negative the function moves to the left. Just be careful because the standard form has a – sign in front of the h, so when you see a positive number in h's place, that means it was a negative value for h. Lastly, a positive k means a shift upward, and a negative k means a shift downward.

In the world of quadratic functions, the standard form is $f(x)=a(x-h)^2+k$, meaning if I had $f(x)=-2(x+2)^2-3$, I would have a standard parabola that has been vertically stretched, flipped upside down, moved to the left by 2 units, and moved down by 3 units. These translations only work in reference to another function, so the problem must provide a baseline, or you need to know the "mother function" (AKA the bare-bones, run of the mill version of that type of function, like a basic parabola for $f(x)=x^2$).

As for function properties, we should be paying attention to four in particular on the SAT. We already discussed domain/range in the Linear Equations and Functions section, so the three remaining are maximum/minimum values, end behavior, and symmetry. All of these are useful in graphing a polynomial function.

Maxima and Minima

The maximum and minimum values for functions are always the greatest (for maximum) and least (for minimum) y coordinate values that exist on the function. To be clear, a maximum is not a point. It is a y-value. The vertex on a parabola is probably the easiest way to picture this topic. If the parabola points up, then the y-value of the vertex is the function's minimum, and if the parabola points down, the vertex's y-value is the maximum.

End Behavior

A function's end behavior is a description of which direction (towards positive or negative infinity) the function moves in the y direction at each end (left and right) of the graph. For example, a standard $y=x^2$ parabola has positive end behavior in both directions.
We can predict a functions end behavior based on both the highest power (whether it is odd or even) on the independent variable (usually x) and the sign (whether it is positive or negative) of the coefficient on that variable.

Highest Power	**Leading Coefficient**	
	Positive	**Negative**
Even	Right and left to +∞	Right and left to -∞
Odd	Left to -∞ and right to +∞	Left to +∞ and right to -∞

Symmetry

A function is said to have symmetry if we find that two halves of the function can be reflected onto one another across an axis of symmetry. If we draw the graph of $y=x^2$, for example, we see that if I fold the paper along the y-axis, then the two halves of the function match each other perfectly. This tells me that the line $x=0$ (the y-axis) is the axis of symmetry for this function. Other functions can have different axes of symmetry. Continuing with parabolas, we always have an axis of symmetry at the x-coordinate of the vertex. In $y=(x-2)^2$ for example, we have an axis of symmetry at $x=2$.

***Practice Questions*:**

1) Line *k*, $y=2x+4$, is reflected about the y-axis and then shifted vertically upward by 3 units. What is the equation of the new line?

2) $f(x)=2(x-3)^2-2$. If g(x) is the translation of f(x), such that g(x) is shifted 2 units up and 4 units to the right, what is the value of c in the point (2,c) on g(x)?

3) Identify the maximum or minimum, end behaviors, and symmetry of the following function: $f(x)=-x^2-4x-4$.

RATIONAL EQUATIONS & FUNCTIONS

Rational equations, or equations with x terms in both the numerator and denominator (frequently on both sides of the equals sign), are fairly straightforward but do require mastery of factoring. If you are comfortable with that skill, rational equations should be a piece of cake.

For example, take the following equation:

$$\frac{x^2+3x+2}{x^2-4x-12} = \frac{4}{x^2-8x+12}$$

If we factor all of the polynomials, we get:

$$\frac{(x+2)(x+1)}{(x+2)(x-6)} = \frac{4}{(x-6)(x-2)}$$

We see that (x+2) appears in the numerator and denominator of the left hand side, meaning we can cancel that term out completely. We then multiply both the (x-6)(x-2) from the denominator on the right over to the left hand side (thereby cancelling it on the right, in the same way you would cancel out a numerical term by multiplying both sides), and find that now the (x-6) term appears in both the numerator and denominator of the left hand side, meaning we can cancel those. This leaves us with

$$(x+1)(x-2)=4$$

Now we must FOIL to find x^2-x-2=4. If we subtract 4 from both sides, that leaves us with x^2-x-6=0. When we re-factor, we get (x+2)(x-3)=0, meaning x=-2,3. Like I said, a facility with factoring is key to these equations.

So far, we've seen plenty of functions with no variables in the denominator. With rational functions, that all changes.

Recall that having 0 in the denominator tends to make mathematicians quite cranky. A fraction is a part over a whole, so if we put 0 in the denominator, we're saying our whole is nothing. You can't have any parts of nothing. Now, imagine that we have a function with a variable, like x, in the denominator. There may be an x value that makes the denominator 0. But that can't happen, right? As a result, the function simply does not exist at that x-coordinate. We signify this fact by drawing a dashed vertical line, called a vertical asymptote, at the given x-value.

How do we determine what values for x will produce a vertical asymptote? The first step is to factor as completely as possible. Remember, with factoring, we can solve for the "zeroes," which turns out to be our goal exactly. Factoring allows us to quickly determine these values, although you may occasionally need to use the quadratic formula. It is important to note that not all factors in the denominator will produce a vertical asymptote. Read on to learn about holes.

In addition to vertical asymptotes, we also frequently have horizontal asymptotes, which describe trends as x becomes larger in the negative and positive directions, in rational equations. These are more guidelines as opposed to the strict rule that vertical asymptotes are. Functions can theoretically cross through a horizontal asymptote. Not so in vertical asymptotes. Prior to factoring, we want to make sure we write our terms in descending degree (terms with highest exponent on the variable go first). There are three possible outcomes:

a) degree of the numerator > degree of denominator
b) degree of numerator = degree of denominator
c) degree of numerator < degree of denominator

In (a), y=0 (the x-axis) is the horizontal asymptote. In (b), the horizontal asymptote is found by dividing the coefficient on the highest degree term in the numerator by the coefficient on the highest degree term in the denominator. In (c) there is no horizontal asymptote, usually a slant/oblique asymptote. We will not be discussing these slant/oblique asymptotes as they are unlikely to show up on the SAT and require an understanding of polynomial long division.

As is the case in rational equations, you will be tempted to factor the numerator and denominator to eliminate any terms that appear in both. This is a good instinct. There's just one wrinkle. If this occurs, it produces a hole at the x-coordinate for that "zero" or factor. This is because the original function still had that factor as part of the denominator. It doesn't have a strong enough hold to be considered a vertical asymptote, but the function still does not exist at that point as originally written.

Now that we've determined where the function doesn't exist, let's talk about how it should be graphed. The first step, as in most equations with exponents, is to find any intercepts. The x-intercepts occur at the "zeroes" of the numerator. After all those values make the function equal to zero, meaning that's where the function crosses the x-intercept. For the y-intercept, just plug in 0 for all of the x's. Next pick a few x-values (ideally ones near the intercepts and asymptotes) to find the corresponding y-values for, and plot them. We should now have enough of a skeleton of a graph to fill in, and voila! You've graphed a rational function.

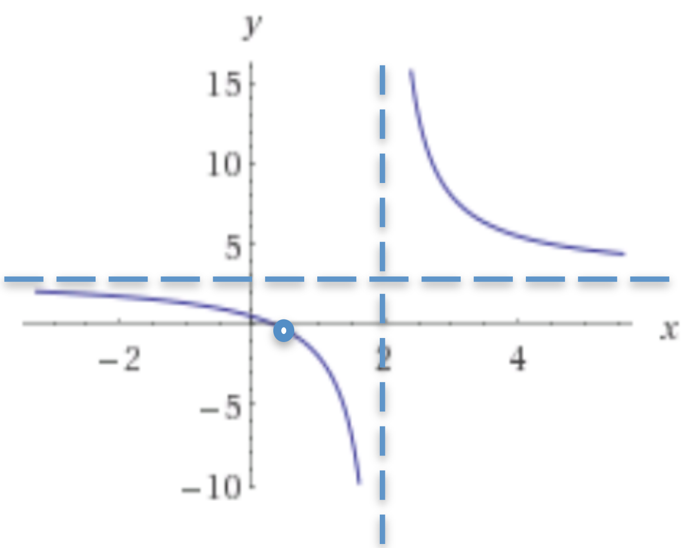
For the SAT, then, it's important that we remember the rules for asymptotes, holes, and finding intercepts. If you have those down, it's simple enough to complete a process of elimination on any question asking for the correct graph or function.
Putting it all together, then, what can we tell about the following function:

$$f(x)= \frac{3x^2-4x+1}{x^2-3x+2}$$

The first thing we notice is that the numerator and denominator have the same degree, meaning we're dealing with a horizontal asymptote at a place other than y=0. By dividing the coefficients of the highest degree terms, we find y=3 to be our horizontal asymptotes. Now we can factor the numerator and denominator to see if there are any holes/help with intercepts/determine the vertical asymptote. Factored the function looks like:

$$f(x)= \frac{(3x-1)(x-1)}{(x-1)(x-2)}$$

We notice that (x-1) appears in the numerator and denominator, meaning we can cancel the two, but must remember there is a whole at x=1 on whatever function we wind up with. We're now left with f(x)=(3x-1)/(x-2). This means there is a vertical asymptote at x=2. As for x-intercepts, we notice that the "zero" in the numerator is x=1/3, producing an intercept at (1/3,0). Finally for the y-intercept, when we plug in 0 for x, we get f(0)=(-1)/(-2), or 1/2. Therefore, our y-intercept is at (0,1/2). Filling in a few points, we get a graph that looks like:



Practice Questions:

1) Solve for x. $\frac{x^3+7x^2+12}{x^2+6x+8} = \frac{20}{(x^2-5x-36)/x}$

2) Graph $f(x)= \frac{2x^2-3x+1}{x^2+6x+8}$

CONIC SECTIONS

We call a particular family of functions conic sections because their shapes can be made by taking a slice through a hollow cone. Each has its own standard form and vocabulary. You should be able to recognize these functions on a given exam.

Parabola

A parabola, as we've previously seen, has a standard form of

$$f(x)=a(x-h)^2+k.$$

We've previously talked about what a, h, and k do in terms of translating, but it is important to note that this standard form is also called the vertex form for a reason.

(h,k) is the vertex of the given parabola.

Recall that a vertex is the maximum or minimum point (depending on if the parabola has a ∩ shape or a ∪ shape, respectively) of the parabola.

The other way to quickly find the vertex if the function is not written in vertex form (e.g., if it is written as $y=ax^2+bx+c$) is to know that:

the x-coordinate of the vertex is equal to –b/2a.

All that that remains is to plug in that that x value in the original function to find its corresponding y value, and we have the vertex point.

Lastly, it becomes simple to find the x-intercepts of a parabola when we know how to factor or use the quadratic formula because

The x-intercepts of a parabola are found at the zeroes of the quadratic.

Circle

The equation for a circle has the standard form of

$$(x-h)^2+(y-k)^2=r^2$$

Notice how we're not using the f(x) notation and that I did not call this a function. Circles, ellipses, and hyperbolas are not classified as functions because they do not pass the Vertical Line Test (draw a vertical line at any x-value and it should only intersect with a function once, otherwise the graph is not of a function).

The center of the circle is at (h,k).
The radius of the circle is r.

Unfortunately, the College Board does not always feel kind enough to give you the circle equation in the standard form. For instance, $x^2+8x+y^2+2x-8=0$ is actually an equation for a circle. To rewrite it in standard form (which gives us easy access to the center coordinates and radius), you may need to employ a technique called Completing the Square.

I know. We're talking about circles. Where does the Square come in? It has to do with the $(x-h)^2$ and $(y-k)^2$ parts of the circle equation. We're trying to rewrite the equation in such a way as to make these terms that are squared! To do so, we need to add numbers to both sides of the equation (if I add the same number to both sides, I'm not changing the value in any way) that make this feasible. Here is the 5-step method that will allow you to re-write the equation.

Step 1: Move any constants to one side of the equation.
Step 2: Rewrite the equation, leaving a blank spot after the x and y terms.
Step 3: Fill in the blank spots with the value that will complete the square (determined by dividing the coefficient on the x^1 and y^1 terms, respectively, by 2 and squaring that quotient).
Step 4: Add the numbers you supplied to fill in the blank spots to the right side of the equation.
Step 5: Factor, looking at the first three terms and second three terms individually.

Let's complete these steps with the equation, $x^2+8x+y^2+2x-8=0$, I mentioned earlier.

Step 1: $x^2+8x+y^2+2x=8$
Step 2: x^2+8x+ ___ $+y^2+2x+$ ___$=8$
Step 3: $x^2+8x+16+y^2+2x+1=8$
Step 4: $x^2+8x+16+y^2+2x+1=7+16+1$
Step 5: $(x+4)^2+(y+1)^2=25$

We can now confidently say we have a circle with a radius of 5 whose center is located at (-4,-1).

Practice Questions:

1) A circle in the (x,y) plane has a center of (6,-4) and an area of 9π. Write the equation for this circle.

2) Find the vertex and the x-intercepts of the following function: $f(x)= x^2-4x+4$.

3) Find the area and center of a circle, the equation of which is $x^2+6x+y^2-4x-3=0$.

DIRECT & INVERSE VARIATION

Whenever you see the phrase "directly proportional" OR "inversely proportional," your immediate reaction should be to set up the equation y=kx (for directly proportional) or y=k/x (for inversely proportional. Usually the key will be to first solve for k, then in the second half of the problem you will use the value you found for k to help you find the missing variable.

Practice Questions:

1) If y is directly proportional to x and y=27 when x=7, what is the value of x when y=54?

2) If y is inversely proportional to x^2 and y=1/2 when x=2, what is the value of x when y=1/8?

SOLVING & GRAPHING INEQUALITIES

Each of the following inequality symbols means something different

> $>$ means greater than
> $<$ means less than
> $\geq$ means greater than or equal to
> $\leq$ means less than or equal to

We tend to be interested in applications of inequalities that involve variables. When you see an inequality express that uses x, for example $2x+2\geq4$, you simply treat the inequality symbol as if it were an equals sign and solve for x as we usually do with equations. In our example, we would subtract 2 from both sides and then divide both sides by 2 to end up with $x\geq1$.

There is one big exception, however. Whenever we divide or multiply both sides by a number, we have to flip the inequality. So if we start with a $\geq$, it becomes a $\leq$ and vice versa. So in an example like $-2x-2\leq6$, we would first add 2 to both sides and then divide both sides by -2. In doing so, we wind up with $x\geq-4$. To graph a solution to an inequality with one variable, we use a number line. In this example we would mark a closed circle at -4 (because it is $\geq$ rather than simply $>$, which would require an open circle), and draw a bolded line in the right hand direction, because we are looking at numbers greater than -4. You would draw that line to the left for a $\leq$ or $<$ symbol.

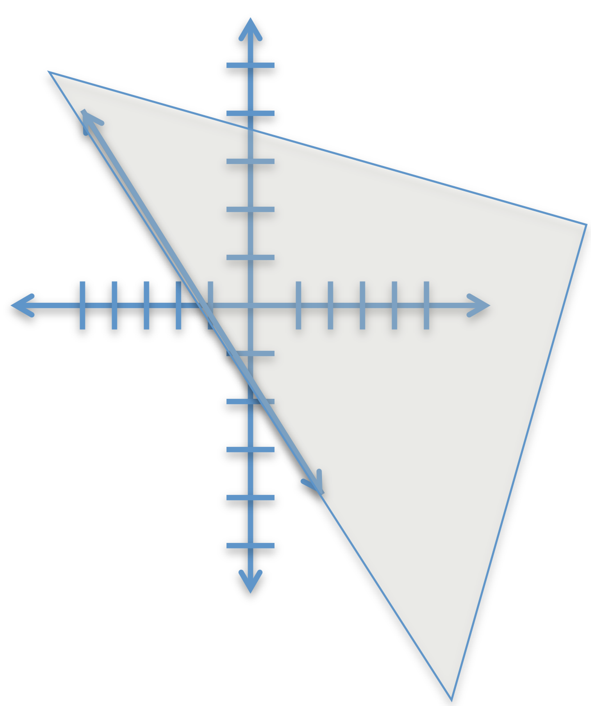
We can also have an inequality involving two variables, usually y and x, which we can treat as a function. For example in the inequality

$$2x-3y\leq4$$

can be rewritten (by subtracting 2x from both sides and then dividing both sides by -3) as

$$y\geq\frac{2}{3}x-\frac{4}{3}$$

To solve for the set of all pairs that satisfy this relationship, we can graph the inequality as if it were a function. We would draw it as a solid line because it is $\geq$, rather than just $>$, which would require the function be drawn with a dashed line. Then because we are looking for all y values greater than that line, we would shade the area above the line.



If the inequality used one of the less than symbols, we would shad the area beneath the line.

Sometimes you will be given a system of inequalities with different signs. In that case you simply graph both and where their shaded portions overlap, you'll find the solution set.

***Practice Questions*:**

1) Which of the following values satisfies the inequality $-2x<0$

   I. all negative numbers
   II. 0
   III. all positive numbers

a) I only
b) II only
c) III only
d) I and II

2) If $y-x<0$ and $x<4$, which of the following points are in the solution set?

   I. (1,2)
   II. (0,0)
   III. (3,1)

a) I only
b) II only
c) III only
d) I and II

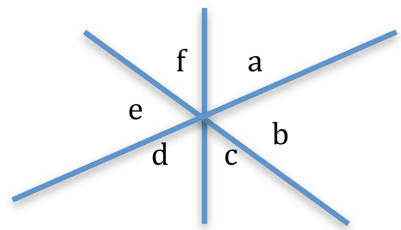
INTERSECTING LINES & TRANSVERSALS

When two lines intersect, the form two pairs of congruent angles. The congruent pairs are what we call vertical angles. Vertical angles are not limited, however, to when two lines intersect. A dozen lines can intersect, and we can still find vertical angles.

Additionally, when two or more angles eventually sum up the top or bottom of a straight line, we call those angles supplementary and they add up to 180°.

For example, in the figure to the right, the following pairs of angles are vertical angles:

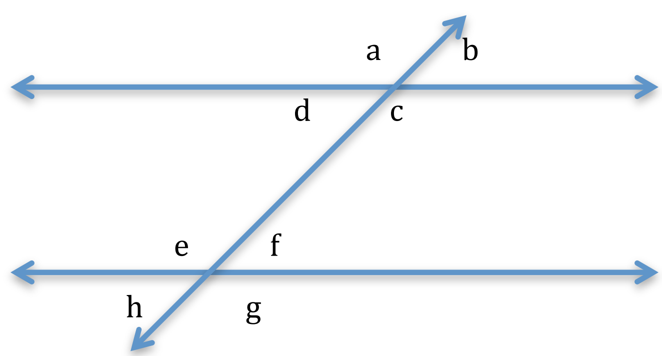
a and d
f and c
b and e



Additionally, e, f, and a come together to form a straight line and are thus supplementary. So are d, c, and b. you can pair any three of these angles moving in a circular direction (clockwise or counterclockwise) and they will be supplementary.

Lastly, with any number of intersecting lines, the sum of all of the angles formed by those intersections is 360°.

A special set of intersecting lines is called a transversal. Transversals require that at least two of the lines be parallel and another line must intersect both of the parallel lines like so:

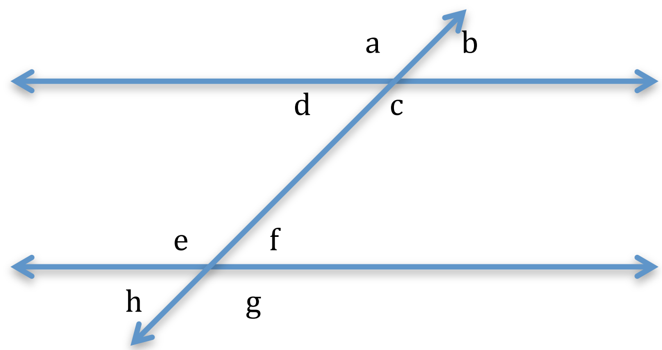


Because the transversal line (the one that cuts through the two parallel lines) is travelling with a constant slope, we know that it must hit two lines that are parallel at exactly the same angle. That means the angles that are formed by the intersections must correspond in some way. We know of course that a is congruent to c, b to d, e to g, and f to h by vertical angles, but there are more congruencies here. a is congruent to e, b to f, c to g, and d to h, as well. We can sort of view these as two separate of intersections, both with the same angles in the same positions (e.g., both a and e are in the upper left corner of their respective intersections, etc.).

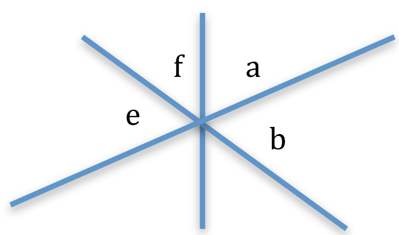
Just a quick note on geometry notation. If you see two letters with a line over them without an arrow on either side, that's referring to a line segment (a distance measured precisely between two points). If the line above has one arrow, that is a ray, which starts definitively at the point symbolized by the first letter and goes on infinitely in the direction of the point symbolized by the next letter. Finally, you could see a pair of letters with a line above them that has two arrows. This is a true line that extends infinitely in both directions (along the two points).

Practice Questions:

1) If angle a is 75°, find the remaining angles b through h.



2) If the two unlabeled angles in the figure have a sum of 25°, what must the sum of e+f+a+b equal?



INTERIOR ANGLES OF A POLYGON

The formula for the sum of the interior angles of a polygon (a many-sided shape) of n sides is

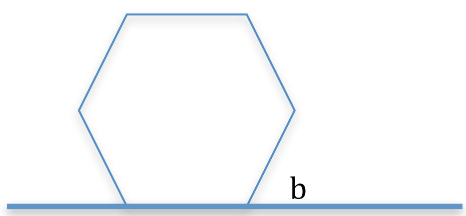
$$(n-2)180°$$

To find the measures of each angle of a regular (all sides and angles congruent) n-sided polygon, we simply divide that formula by n.

$$\frac{(n-2)180°}{n}$$

Practice Questions:

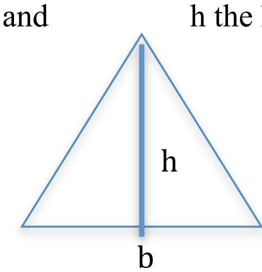
1) A regular hexagon is drawn atop a line as shown below. What is the measure of angle b?



2) A student walks north across campus until reaching the library, at which point he turns right walking until he passes his friends dorm. At the dorm he hears of a study group taking place at the dining hall, so he heads southeast. At the dining hall, he hears that the group moved to the quad to the northwest. After studying he returns to his apartment. What is the sum of the interior angles of the shape bounded by the student's walking path?

TRIANGLES

A triangle is an area enclosed by three lines or segments, which form three vertices. The interior angles of a triangle must sum up to 180°, and the area of the triangle can be found using the formula A=½bh, where b signifies the length of the base and h the height. The perimeter is simply the sum of all the sides



Before we move on to the types of triangles that exist, we should note the Triangle Inequality Theorem, which states that in the event of a triangle possessing a largest side, this largest side must be greater than either of the other two (obviously) and smaller than the sum of the other two (less obvious). If you take two pens and create an angle with them, you can imagine the space between the pens as joined by a third, imaginary side. If you increase the angle created by the pens, you can imagine the third side getting longer and longer. Eventually, though, the third side is so long that the two pens are now joined in a straight line. We no longer have a triangle. That is why the third and largest side must always be smaller than the sum of the other two.

There are basically **five special triangles**.

The first is the equilateral triangle in which all of the sides and all of the angles are congruent. Because we know the total interior angles add up to 180°, it should be simple for us to deduce that each of those angles would then be 60° (180/3).

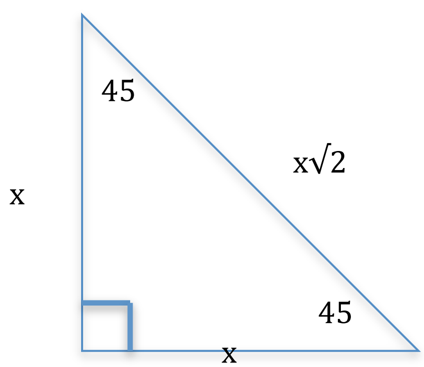
The second is the isosceles triangle in which two of the sides are congruent, and the angles opposite those sides are also congruent to one another. This is a good moment to say that the angles of a triangle are roughly proportional to the side opposite those angles. What we mean is that the biggest angle will always be opposite the biggest side, the smallest angle opposite the smallest side, etc.

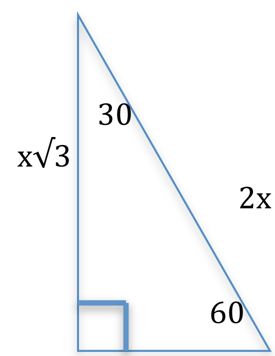
The third special triangle is the right triangle. The right triangle is unique in that one of its sides is also its height, and the side lengths can be determined using the Pythagorean Theorem.

There are also two special right triangles.

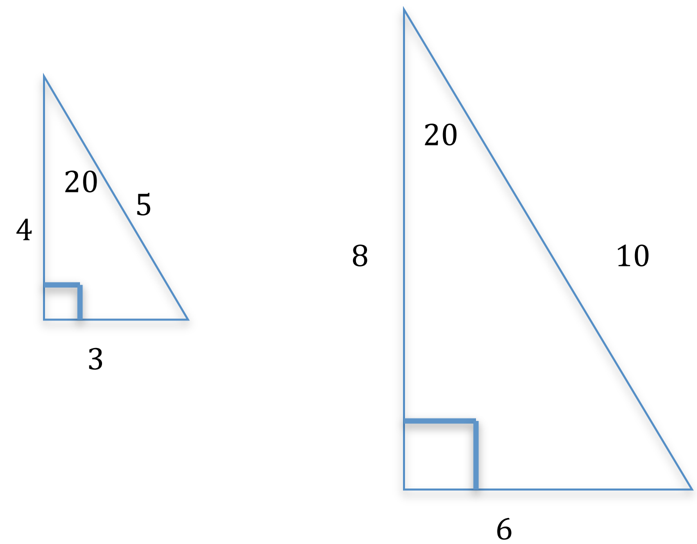
The first is an isosceles right triangle, which we call a 45-45-90 right triangle. The two sides opposite the 45° angles are congruent, and we tend to call their length some number, x. The hypotenuse of that triangle will always be $x\sqrt{2}$, in that case.

The second is what we call a 30-60-90 right triangle based on each of the three angle measures. The smallest side, which we call x, is opposite the smallest angle, 30°. The second largest side, $x\sqrt{3}$, is opposite the second largest angle, 60°, and the largest side, 2x, is opposite the 90° angle.

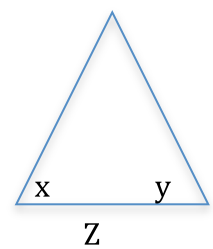




Finally, triangles can be similar to one another in addition to any sort of congruency. Similar triangles have the same angle measurements and the same ratio for their side lengths. In the two figures below, we see an example of similar triangles. Note that each of the side lengths of the triangle on the right is double its corresponding length in the other triangles.

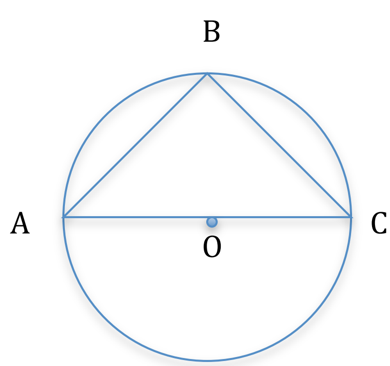


Finally, we need to consider when we can claim that triangles are congruent to one another. You may remember the following acronyms from your geometry class: SSS, ASA, SAS, and AAS, where S stands for a side and A for an angle. The key to these congruencies is that they must occur in the order of the acronym. For example, in ASA, it's not just that any two angles and a single side must be congruent from one triangle to another, it's that (as shown) an angle (like x) followed by a side (like Z) followed by a different angle (like y) must be congruent between the two triangles.

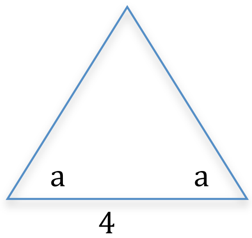


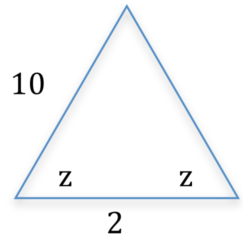
Practice Questions:

1) If the area of triangle ABC is 36, then what is the area of the circle with center O.



2) Find the perimeter of the following triangles, if a=60°. Are the triangles similar? Why or why not?

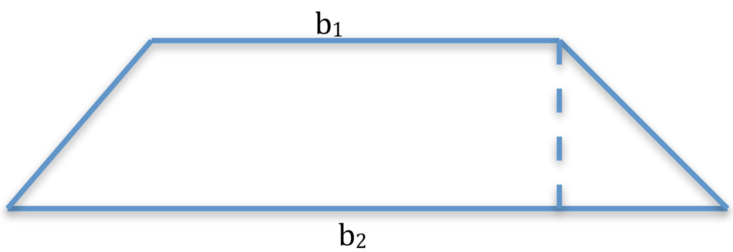
i) 

ii) 

TRAPEZOIDS & PARALLELOGRAMS

A trapezoid has one pair of parallel sides joined together by two non-parallel, non perpendicular sides. We call the two parallel sides the bases, b_1 and b_2. The distance between the two parallel sides is called the altitude, or height. From these quantities we can determine the area of a trapezoid to be

$$A=\frac{b_1+b_2}{2} \times h$$

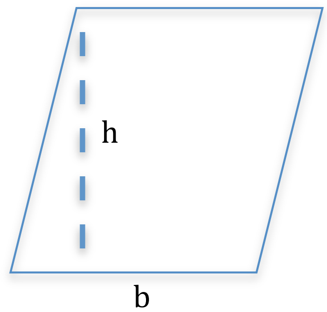


Don't worry too much if you don't remember the formula, though, because the trapezoid is really just two right triangles and rectangle, when you think about it. The dimensions of the rectangle are b_1xh and the two triangles each have a height of h. The triangles' bases will add up to the difference in length between the two bases of the trapezoid. Or, you know, you could just make sure you memorize the formula

As a side note, when the two slanted sides of a trapezoid are congruent, we call the shape an isosceles trapezoid.

A parallelogram is a four-sided shape with two pairs of parallel, congruent sides. The congruent, parallel sides are always opposite one another.

The basic formula for the area of a parallelogram is A=bh



Rectangles, rhombi, and squares are all examples of parallelograms.

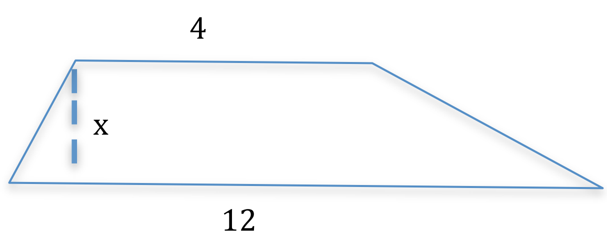
A rectangle has four 90° angles, and its height is one of its sides. That makes its area formula is A= l x w.

A rhombus is a parallelogram with four congruent sides, but the angles may not all be 90°. That means its height is not equal to its base, and you'll need to use the standard parallelogram area formula.

A square is a parallelogram with four congruent sides and four 90°. Since its height is one of the sides and all the sides are congruent, a square's area formula is A= s x s = s^2, where s stands for side length.

Practice Questions

1) Find x if the area of the trapezoid is equal to $4x^2$.



2) Find the diagonal of a square with area of 64.

CIRCLES

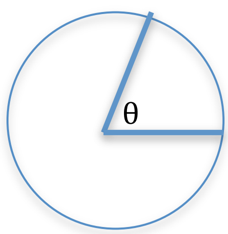
Circles are some of the most tested on shapes in any standardized test. First, realize that a circle is a shape drawn by collecting all of the points equidistant from a center on a plane. We call the distance from any of these points to the edge of the circle a radius. The total length of the distance around a circle is called the circumference, the formula of which is

$$C=2\pi r=\pi d$$

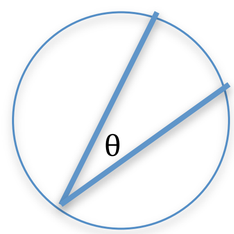
where r is the radius and d is the diameter the length from one edge of the circle through the center to another edge). The diameter divides the circle into two semicircles. The diameter is the longest possible cord, or a line segment connecting two points along the circumference.
Any line, segment, or ray that touches the edge of the circle only once is said to be tangent to the circle. The entire shape is a 360° around, which is why we can measure any portion (called an arc length) of the circumference, in either degrees as a proportion of the circle or in absolute length (like meters or feet). The area of a circle is given by the formula

$$A=\pi r^2$$

Measuring an arc length can be quite useful. It is easiest when the arc is either cut by an inscribed or central angle. We use the Greek letter θ as the variable for an angle.



Central Angle



Inscribed Angle

If the arc length is cut by a central angle, its degree measure is the same as the measure of the central angle. We can then use the proportion of that degree measure out of 360° and relate it to the proportion of the arc length divided by the circumference.

$$\frac{\text{Arc Measure (in degrees)}}{360^\circ}=\frac{\text{Arc Length}}{\text{Circumference}}$$

If the arc length is cut by an inscribed angle, its measure is twice that angle. We can then use the same relationship above to determine the length.

If the central angle is given in radians instead of degrees, the given problem can be even simpler. We can solve either for the radius or arc length (depending on which of the two is given) with the following formula, where s is the arc length in radians, r is the radius, and θ is the central angle in radians:

$$s=r\theta$$

The portion of the circle marked off by the two radii that extend from the center to form a central angle is called a sector. The area of that sector can be determined by looking at the ratio of the central angle out of 360° and using a proportion to compare it to the ratio of the sector area out of the total area of the circle.

$$\frac{\text{Arc Measure (in degrees)}}{360°}=\frac{\text{Sector Area}}{\text{Circle Area}}$$

***Practice Questions*:**

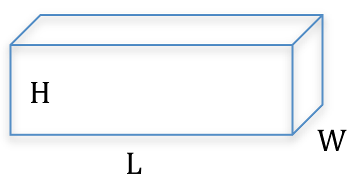
1) Find the radius of the circle if the sector formed by the radii marking off a central angle of 30° is 2π.

2) What is the length of an arc formed by the cords that inscribe an angle of 45° in a circle with a radius of 8?

VOLUME

We will consider four 3-D shapes: rectangular prisms, cylinders, and cones, and spheres.

Rectangular Prisms



The volume of a rectangular prism (and many shapes that can be uniformly cut into 2-D slices of uniform area) is the area of its base times its height, or

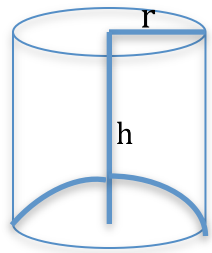
$$V=LxWxH$$

Recall that a square is just a special type of rectangle, and you will see that cubes fit into this category as well, except all of the lengths are the same, making the formula for the volume of a cube

$$V=s^3$$

Cylinders

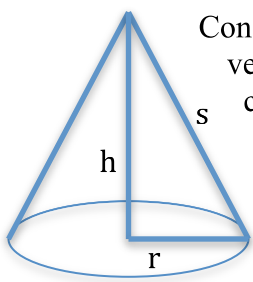
A cylinder has two congruent circular bases capping a curved/"rolled up" rectangle.



In the same way that the rectangular prism's volume was the area of the base times the height, the volume of a cylinder is the area of the cicular base times the height or

$$V=\pi r^2 h$$

Cones



Cones are unique shapes with a circular base and a curved edge which meets at a single vertex. The length traveling down the edge from the vertex to the circular base is called the slant height, or s. The distance from the vertex travelling straight down to the center of the circular base is the height, or h. And the radius of the circular base is abbreviated r. With this in mind the two major formulae are

$$V=(1/3)\pi r^2 h$$

Sphere

The 3-D projection of a circle is a sphere. Spheres have a radius that extends from the shape's center to any point along the edge of the sphere.

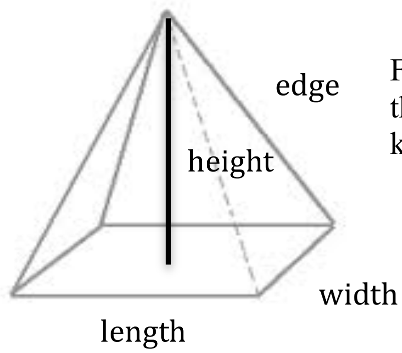
The two important formula to remember is

$$V=(4/3)\pi r^3$$

Keep in mind that it is not uncommon for problems to have shapes to be inscribed in a sphere, in the same way 2-D shapes are often inscribed in a circle.

Pyramid

We've all seen (at least pictures of) the Great Pyramids of Giza. That's pretty much the shape you'll need to keep in mind to understand the way the SAT tests pyramids.



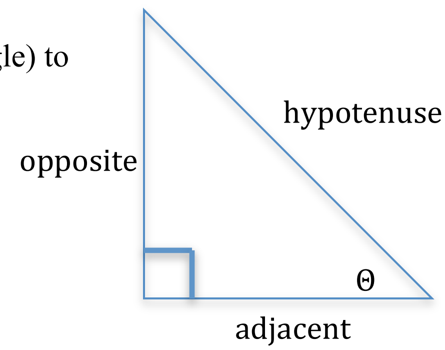
For a pyramid V=(1/3)*lwh*, where l is length of base, w is the width of the base, and h is the height, which is frequently determined by knowing the edge length and constructing triangles to solve.

Practice Questions

1) What is the volume of the largest sphere that can be inscribed in a cube with a volume of 8?

2) What is the volume of a pyramid with an edge length of 5 and a square base of side length 3√2?

BASIC RIGHT TRIANGLE TRIGONOMETRY



In any right triangle, if we specify one angle (besides the right angle) to be Θ, we can name the sides relative to that angle. One leg will be opposite the angle, another will be adjacent to it, and the hypotenuse will always be opposite the 90° angle.

Recognizing these relationships we can evaluate ratios of side lengths relative to the angle's position, or in other words, do trigonometry.

The sine of an angle is the opposite over the hypotenuse, while the cosine is the adjacent over the hypotenuse. The tangent, though, is the opposite over the adjacent.
In their abbreviated forms they look like:

$$\sin\Theta=\frac{O}{H}$$

$$\cos\Theta=\frac{A}{H}$$

$$\tan\Theta=\frac{O}{A}=\frac{\sin\Theta}{\cos\Theta}$$

The great mnemonic device to help you remember these relationships is SOH-CAH-TOA for Sine=Opposite/Hypotenuse, Cosine=Adjacent/Hypotenuse, and Tangent=Opposite/Adjacent.

These relationships are incredibly useful, and there is one last thing that you might find helpful about them. Let's say we know two of the side lengths of a right triangle, but want to know one of the angles. The trigonometric ratios allow us to find this angle, by taking the inverse sine, cosine, or tangent (written as $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, respectively) of the appropriate side length ratio. For instance, let's say I knew that the side opposite my angle of interest was 4 and the side adjacent to the same angle was 3. I know that the $\tan\Theta=4/3$, but not Θ, itself. Now I can take the $\tan^{-1}(4/3)$ and that equals Θ. We can find that value by using our calculators. On a TI calculator you will hit the Second button followed by the appropriate trigonometric function.

Practice Questions

1) A flagpole casts a shadow of 14ft. Given an angle of elevation from the ground at the edge of the shadow to the top of the pole of 30°, how tall is the flagpole?

2) At what angle will a ramp be if I only have 10 feet horizontally to travel 5 feet vertically? How long is the ramp?

RADIANS

For ACT purposes, a radian is just another way of talking about a degree measure (longer discussions of radians are more suited to the SAT II Math exam).

To convert between the two, we should know that a 2π radian measure is the equivalent of 360°. That means we can always set up a proportion using that relationship.

$$\frac{360°}{2\pi} = \frac{x \text{ degrees}}{y \text{ radians}}$$

These radian measures actually derive from the circumference of a circle with a radius of 1, which would create a circumference equal to 2π.

***Practice Questions*:**

1) What is one possible value of Θ in radians if $\sin\Theta = 1/2$?

2) What is 270° in radians?

ESSAY SECTION

The essay on the SAT is an optional 50-minute section available to take when the core test (both Evidence Based Reading and Writing and both Math sections) is complete. When you take the SAT Essay section, you will receive a score that is wholly separate from the overall SAT score.

Further, the essay is scored in three categories: Reading, Analysis, and Writing. Each category is scored on a scale of 2 to 8. These three scores are reported separately and make up your overall Essay component score.

Two trained graders (usually high school teachers) review your essay, and each one assigns a score between 1 and 4 in each of the four categories listed above. These scores are added together to yield your 2 to 8 scale. If the two readers' scores disagree by more than one point (one gave you a 4 and the other gave you a 2), then a third reader is brought in to resolve the difference.

THE THREE CATEGORES

As mentioned above, the SAT essay is scored in three categories. Below is a simple chart of how the SAT described each of these areas:

Reading	A successful essay shows that you understood the passage, including the interplay of central ideas and important details. It also shows an effective use of textual evidence.
Analysis	A successful essay shows your understanding of how the author builds an argument by: - Examining the author’s use of evidence, reasoning, and other stylistic and persuasive techniques - Supporting and developing claims with well-chosen evidence from the passage
Writing	A successful essay is focused, organized, and precise, with an appropriate style and tone that varies sentence structure and follows the conventions of standard written English.

THE SCORING RUBERIC

Here is the actual scoring rubric that the graders use when reading your essay.

The SAT Writing Test Scoring Rubric

Score	Reading	Analysis	Writing
4	**Advanced:** The response demonstrates thorough comprehension of the source text. The response shows an understanding of the text's central idea(s) and of most important details and how they interrelate, demonstrating a comprehensive understanding of the text. The response is free of errors of fact or interpretation with regard to the text. The response makes skillful use of textual evidence (quotations, paraphrases, or both), demonstrating a complete understanding of the source text.	**Advanced:** The response offers an insightful analysis of the source text and demonstrates a sophisticated understanding of the analytical task. The response offers a thorough, well-considered evaluation of the author's use of evidence, reasoning, and/or stylistic and persuasive elements, and/or feature(s) of the student's own choosing. The response contains relevant, sufficient, and strategically chosen support for claim(s) or point(s) made. The response focuses consistently on those features of the text that are most relevant to addressing the task.	**Advanced:** The response is cohesive and demonstrates a highly effective use and command of language. The response includes a precise central claim. The response includes a skillful introduction and conclusion. The response demonstrates a deliberate and highly effective progression of ideas both within paragraphs and throughout the essay. The response has a wide variety in sentence structures. The response demonstrates a consistent use of precise word choice. The response maintains a formal style and objective tone. The response shows a strong command of the conventions of standard written English and is free or virtually free of errors.
3	**Proficient:** The response demonstrates effective comprehension of the source text. The response shows an understanding of the text's central idea(s) and important details. The response is free of substantive errors of fact and interpretation with regard to the text. The response makes appropriate use of textual evidence (quotations, paraphrases, or both), demonstrating an understanding of the source text.	**Proficient:** The response offers an effective analysis of the source text and demonstrates an understanding of the analytical task. The response competently evaluates the author's use of evidence, reasoning, and/or stylistic and persuasive elements, and/or feature(s) of the student's own choosing. The response contains relevant and sufficient support for claim(s) or point(s) made. The response focuses primarily on those features of the text that are most relevant to addressing the task.	**Proficient:** The response is mostly cohesive and demonstrates effective use and control of language. The response includes a central claim or implicit controlling idea. The response includes an effective introduction and conclusion. The response demonstrates a clear progression of ideas both within paragraphs and throughout the essay. The response has variety in sentence structures. The response demonstrates some precise word choice. The response maintains a formal style and objective tone. The response shows a good control of the conventions of standard written English and is free of significant errors that detract from the quality of writing.

2	**Partial:** The response demonstrates some comprehension of the source text. The response shows an understanding of the text's central idea(s) but not of important details. The response may contain errors of fact and/or interpretation with regard to the text. The response makes limited and/or haphazard use of textual evidence (quotations, paraphrases, or both), demonstrating some understanding of the source text.	**Partial:** The response offers limited analysis of the source text and demonstrates only partial understanding of the analytical task. The response identifies and attempts to describe the author's use of evidence, reasoning, and/or stylistic and persuasive elements, and/or feature(s) of the student's own choosing, but merely asserts rather than explains their importance, or one or more aspects of the response's analysis are unwarranted based on the text. The response contains little or no support for claim(s) or point(s) made. The response may lack a clear focus on those features of the text that are most relevant to addressing the task.	**Partial:** The response demonstrates little or no cohesion and limited skill in the use and control of language. The response may lack a clear central claim or controlling idea or may deviate from the claim or idea over the course of the response. The response may include an ineffective introduction and/or conclusion. The response may demonstrate some progression of ideas within paragraphs but not throughout the response. The response has limited variety in sentence structures; sentence structures may be repetitive. The response demonstrates general or vague word choice; word choice may be repetitive. The response may deviate noticeably from a formal style and objective tone. The response shows a limited control of the conventions of standard written English and contains errors that detract from the quality of writing and may impede understanding.
1	**Inadequate:** The response demonstrates little or no comprehension of the source text. The response fails to show an understanding of the text's central idea(s), and may include only details without reference to central idea(s). The response may contain numerous errors of fact and/or interpretation with regard to the text. The response makes little or no use of textual evidence (quotations, paraphrases, or both), demonstrating little or no understanding of the source text.	**Inadequate:** The response offers little or no analysis or ineffective analysis of the source text and demonstrates little or no understanding of the analytic task. The response identifies without explanation some aspects of the author's use of evidence, reasoning, and/or stylistic and persuasive elements, and/or feature(s) of the student's choosing, Or numerous aspects of the response's analysis are unwarranted based on the text, The response contains little or no support for claim(s) or point(s) made, or support is largely irrelevant. The response may not focus on features of the text that are relevant to addressing the task. Or the response offers no discernible analysis (e.g., is largely or exclusively summary).	**Inadequate:** The response demonstrates little or no cohesion and inadequate skill in the use and control of language. The response may lack a clear central claim or controlling idea. The response lacks a recognizable introduction and conclusion. The response does not have a discernible progression of ideas. The response lacks variety in sentence structures; sentence structures may be repetitive. The response demonstrates general and vague word choice; word choice may be poor or inaccurate. The response may lack a formal style and objective tone. The response shows a weak control of the conventions of standard written English and may contain numerous errors that undermine the quality of writing.

SAMPLE PROMPT

The prompt for the SAT contains three components. First, you have a short set of directions of what to consider when reading the passage. Next, you are presented with an extended passage taken from a speech or publication of some sort. Finally, you are given an "essay task" which is essentially the prompt you are responding to. The essay format is always the same.

COMPONENT 1

The first component of the prompt never changes. Below is the actual text:

As you read the passage below, consider how the author uses

- evidence, such as facts or examples, to support claims.
- reasoning to develop ideas and to connect claims and evidence.
- stylistic or persuasive elements, such as word choice or appeals to emotion, to add power to the ideas expressed.

This first component is simply giving you a "heads up" about what to pay attention to when reading. As we're going to see on the third component, your essay task is going to ask you about the author's argument. Therefore, this is telling you to specifically look for elements that will help you write your essay. Notice there are three points the prompt is asking you to consider. First is evidence that supports the author's claim, next is the reasoning used to develop the claim, and finally is the writing style used to drive home the point. These are essentially the three elements at an author's disposal when trying to prove their point – they can give solid evidence, make an argument, and add style to finish the argument.

COMPONENT 2

The second component to the prompt is an extended essay or speech. This will come in at a robust 750 words and will be taken from a pretty high-level source. It may be a famous speech or an article taken from a newspaper. Either way, you can be assured that it will be a very "adult" source to the piece. As an example, look at this:

Adapted from Dana Gioia, "Why Literature Matters" ©2005 by The New York Times Company. Originally published April 10, 2005.

[A] strange thing has happened in the American arts during the past quarter century. While income rose to unforeseen levels, college attendance ballooned, and access to information increased enormously, the interest young Americans showed in the arts—and especially literature—actually diminished.

According to the 2002 Survey of Public Participation in the Arts, a population study designed and commissioned by the National Endowment for the Arts (and executed by the US Bureau of the Census), arts participation by Americans has declined for eight of the nine major forms that are measured....The declines have been most severe among younger adults (ages 18–24). The most worrisome finding in the 2002 study, however, is the declining percentage of Americans, especially young adults, reading literature.

That individuals at a time of crucial intellectual and emotional development bypass the joys and challenges of literature is a troubling trend. If it were true that they substituted histories, biographies, or political works for literature, one might not worry. But book reading of any kind is falling as well.

That such a longstanding and fundamental cultural activity should slip so swiftly, especially among young adults, signifies deep transformations in contemporary life. To call attention to the trend, the Arts Endowment issued the reading portion of the Survey as a separate report, “Reading at Risk: A Survey of Literary Reading in America.”

The decline in reading has consequences that go beyond literature. The significance of reading has become a persistent theme in the business world. The February issue of *Wired* magazine, for example, sketches a new set of mental skills and habits proper to the 21st century, aptitudes decidedly literary in character: not “linear, logical, analytical talents,” author Daniel Pink states, but “the ability to create artistic and emotional beauty, to detect patterns and opportunities, to craft a satisfying narrative.” When asked what kind of talents they like to see in management positions, business leaders consistently set imagination, creativity, and higher-order thinking at the top.

Ironically, the value of reading and the intellectual faculties that it inculcates appear most clearly as active and engaged literacy declines. There is now a growing awareness of the consequences of nonreading to the workplace. In 2001 the National Association of Manufacturers polled its members on skill deficiencies among employees. Among hourly workers, poor reading skills ranked second, and 38 percent of employers complained that local schools inadequately taught reading comprehension.

The decline of reading is also taking its toll in the civic sphere....A 2003 study of 15- to 26-year-olds’ civic knowledge by the National Conference of State Legislatures concluded, “Young people do not understand the ideals of citizenship… and their appreciation and support of American democracy is limited.”

It is probably no surprise that declining rates of literary reading coincide with declining levels of historical and political awareness among young people. One of the surprising findings of “Reading at Risk” was that literary readers are markedly more civically engaged than nonreaders, scoring two to four times more likely to perform charity work, visit a museum, or attend a sporting event. One reason for their higher social and cultural interactions may lie in the kind of civic and historical knowledge that comes with literary reading....

The evidence of literature’s importance to civic, personal, and economic health is too strong to ignore. The decline of literary reading foreshadows serious long-term social and economic problems, and it is time to bring literature and the other arts into discussions of public policy. Libraries, schools, and public agencies do noble work, but addressing the reading issue will require the leadership of politicians and the business community as well....

Reading is not a timeless, universal capability. Advanced literacy is a specific intellectual skill and social habit that depends on a great many educational, cultural, and economic factors. As more Americans lose this capability, our nation becomes less informed, active, and independent-minded. These are not the qualities that a free, innovative, or productive society can afford to lose.

COMPONENT 3

The third component is the actual assignment. Here is the one that went with the above essay:

> Write an essay in which you explain how Dana Gioia builds an argument to persuade his audience that the decline of reading in America will have a negative effect on society. In your essay, analyze how Gioia uses one or more of the features in the directions that precede the passage (or features of your own choice) to strengthen the logic and persuasiveness of his argument. Be sure that your analysis focuses on the most relevant features of the passage.

Your essay should not explain whether you agree with Gioia's claims, but rather explain how Gioia builds an argument to persuade his audience.

The first important point here is that the test maker is giving you a very important "tip" in the beginning of the prompt. Notice how the first sentence actually states the author's argument. This is extremely important because it ensures you do not get confused.

The second point is that the prompt is asking you to "analyze" how the author makes their argument. It is being clear that the task is to present the details of how the argument is made.

Finally, the last line makes it clear that you are not to say your own opinion. This is very different from many essays you may have written in the past, where first you analyze and then you pick a side. This essay is meant to be an unbiased presentation of fact.

THE ASSIGNMENT

As you can see from the sample prompt and the grading rubric, your overall goal is to make sure to explain how the author makes their argument. The key thing to remember is that you want to "write to your reader." In other words, make sure you are writing the essay in a way that encourages your reader to give you the highest score possible.

You are being graded in three areas: Reading, Analysis, and Writing. Let's break this down as if there are three "assignments" you are tasked with.

Assignment 1: Your Reading grade is meant to evaluate how well you understood the passage. This is essentially judged based on your ability to use effective and accurate quoting while writing your essay. Everything you say and claim should come with a relevant and appropriate quote.

Assignment 2: Your Analysis grade evaluates how well you understand how the author made their argument. Here you will want to focus on the three critical components from Component 1 above: evaluating evidence the author brought, evaluating the logic and reason they used and finally the style and tone they utilized. Our Tier One Outline below will have you break these into different body paragraphs.

Assignment 3: Your Writing score is based primarily on your ability to properly organize and present your writing. Following the below Tier One Outline will ensure this is done well.

TIER ONE METHOD FOR SAT ESSAY WRITING

Step 1: Read "Component 3" before the actual passage.

Step 2: "Actively" read the passage twice.

Step 3: Create a Tier One Outline.

Step 4: Write your essay closely following the Tier One Outline.

Step 5: Revise your essay.

Before describing each of these steps in depth, let us first address the issue of pacing. There is a lot to read, a lot to write, and total of 50 minutes to complete the assignment. Time management is critical. Therefore, here is the breakdown of how to spend your time:

1) **10 Minutes** should be spent on reading Component 3 as well the passage two times.
2) **5 Minutes** should be spent on creating your Tier One Outline.
3) **30 Minutes** should be spent on writing the essay.
4) **5 Minutes** should be spent revising your essay.

STEP 1: READ "COMPONENT 3" BEFORE THE ACTUAL PASSAGE

Remember that embedded in to the first sentence of Component 3 of your prompt is the actual argument the author is making in the passage. Since your essay is going to be devoted to how the author proves their argument, it is imperative that you understand that argument before you start reading.

STEP 2: "ACTIVELY" READ THE PASSAGE TWICE

Back in the Reading chapter we discussed "active" reading and defined it as reading for a purpose. That was as opposed to "passively" reading, where you take note of whatever strikes you as interesting when reading. Remember that the passage in the essay prompt will be both long and complicated. Therefore, reading with a purpose is going to be key.

You will want to keep the argument in mind that you found from Step 1. Next, you will read each paragraph asking yourself the question "is this proof for the argument or not?" Whenever you come across something that is a proof for your argument, simply put a checkmark at the beginning of the line. There is no need to take notes and no need to underline. Your goal is to read quickly.

Once you have gone through the passage once, you should take a moment to reflect on how well you understood how the author argues his or her point. This should be a quick, 30 seconds or so, exercise just to gauge how well you understood what you read. Then you should go ahead and read the passage a second time. On this second reading, you should make sure to double-check that you have noted all of the proofs in the passage – it should be almost like skimming since you have already read it once.

STEP 3: CREATE A TIER ONE OUTLINE

The Tier One Outline is literally an outline that you will create prior to writing your essay during the exam on your test day. By planning your essay in the prescribed way, you will guarantee that your essay responds to every aspect of the prompt. The grader will be checking for that when reviewing your essay. Of course, that doesn't promise a perfect score, in-and-of-itself, but it certainly sets you up for one.
The key ingredients to our Tier One Outline are the following:

First, you must provide a ***very*** well-worded, ***detailed*** description of the author's argument. Since the grader is looking to make sure you understood the argument, when you first state it, it should be abundantly clear and precise. This requirement differs from a normal outline where you state your thesis. Here there is no thesis. However, you also need to mention briefly the three ways in which the argument was proven: evidence, logic, and writing style.

Second, outline your body paragraphs. There should be a total of three body paragraphs (in some situations it is okay to break up each one into two paragraphs). Each paragraph should explain one of the three strategies that the author used, as you explained above. In the outline itself, you should write each of these strategies out in one sentence. If you can't break it down to one sentence, then you don't understand it and neither will the reader.

Second, you are going to want to give quotes from the text to support what you are arguing. Therefore, in this outline you will put the line reference number for 2-4 quotes that support your statements.

Finally, you write a conclusion paragraph that sums up the author's argument.

TIER ONE OUTLINE

I. Introduction Paragraph

a) Provide a *very* well worded, *detailed* description of the author's argument.
b) Describe the three ways in which you will describe how the argument was developed/proven.

II. Body Paragraph 1

a) Describe how the author uses *evidence* to support their argument.
b) Line reference number for proof 1.
c) Line reference number for proof 2.

III. Body Paragraph 2

a) Describe how the author uses *logic* to support their argument.
b) Line reference number for proof 1.
c) Line reference number for proof 2.

IV. Body Paragraph 3

a) Describe how the author uses *writing style/tone* to support their argument.
b) Line reference number for proof 1.
c) Line reference number for proof 2.

V. Conclusion

a) Recap the author's argument.

ACTIVITY FOR STEP 2:

Directions: Use the sample prompt above and create a Tier One Outline.

I. Introduction Paragraph

a. Author's Argument: ____

____

b. The three ways the author proves their argument:

____

II. Body Paragraph 1

How author proves argument using evidence: ________________________________

Proof 1: ____

Quote that shows the author's argument being proved/supported via evidence:

____

Proof 2: ____

Quote that shows the author's argument being proved/supported via evidence:

____

III. Body Paragraph 2

How author proves argument using logic: ________________________________

Proof 1: ____

Quote that shows the author's argument being proved/supported via logic:

____

Proof 2: ____

Quote that shows the author's argument being proved/supported via logic:

____

IV. Body Paragraph 3

How author proves argument using tone/style: ____

Proof 1: ____

Quote that shows the author's argument being proved/supported via tone/writing style: ____

Proof 2: ____

Quote that shows the author's argument being proved/supported via tone/writing style: ____

V. Conclusion

Recap the author's argument and how they proved it. ____________________________

____

STEP 3: WRITE YOUR ESSAY FOLLOWING THE TIER ONE OUTLINE

To write an effective essay, you need to "fill in the blanks" of your outline; merely regurgitate the structure and ideas from your outline and add some compelling thoughts, filler sentences, and a little bit of good style, and you will have yourself a fantastic essay.

SAY HELLO, GET ACQUAINTED

For the intro paragraph you need to introduce your essay to your grader, provide a strong and provable summary of the author's argument, and introduce the ways they made their case.

PROVE IT, WE DARE YOU!

In each body paragraph, you must present examples of the author's strategies. Most importantly, you need to make sure that you set out to prove your point very strongly throughout the paragraph. Include relevant details, particularly quotes from the original text. Part of providing details means that you must state the obvious – don't assume that the reader knows anything about your topic.

MAKE IT END…PLEASE MAKE IT END!!

In your conclusion, you should try not to make grammar mistakes. Endeavor to make clear points, and use nicely varied vocabulary, if possible. Even if you don't meet these standards throughout the essay, at least go out with a bang and make a good final impression!

Furthermore, you need to make it abundantly clear that your essay was well-structured and that you thoroughly described the ways the the author demonstrated his or her argument. For that reason, now is the right time to quickly re-explain the author's argument.

A FEW ADDITIONAL PIECES OF ADVICE…FOR FREE!

Here are a handful of hints and tricks to help you increase your score.

1) **Essay Length:** Usually, the longer, the better.

2) **Ensure Good Communication:** Make sure to communicate all your main ideas well and use a lot of quotes from the text.

3) **Avoid Personal Qualifiers:** A personal qualifier is a statement like "I think." You need to prove your points, not think them.

4) **Avoid Personal Language:** In general, it is advisable to stay away from words like "I," "me," or "we."

5) **Good Vocabulary:** It is good to employ advanced vocabulary, but you want to make sure that it flows with the essay; do not force words that do not belong into a sentence. Worse than the lack of advanced vocabulary is the misuse of it.

6) **Varied Sentence Structure:** This means that it is good for you to have a mixture of sentences. Some sentences should be compound, some simple; some statements, some questions; some quotations, etc.

7) **Know When to Skip Lines:** You can skip lines between paragraphs but you should NOT skip lines between every line of writing.

8) **If You Get Stuck… :** If you get a case of writer's block, use phrases such as "for example," "on the other hand," and "in other words." Starting a new sentence with one of these phrases will make it much easier to start writing again.

Tier One Tips!

1. Don't forget to follow your Tier One Outline! Remember, the whole point of having one was to help make this step successful.
2. Try to write a stellar conclusion; final impressions matter!
3. If you get stuck with writer's block, just start a new sentence with the phrase "for example" or "in other words" to help get you back on track.

ACTIVITY FOR STEP 3:

Directions: **Use the Tier One Outline from the previous activity to write a sample essay.**

STEP 4: REVISE YOUR ESSAY

THE CHECKLIST

To help with this step we have created a simple checklist of things to look for as you are doing your final step, revising your essay.

1) **Intro:** Make sure that your introduction is clear, to the point, and actually reflects what was proved throughout your essay. Further, the author's argument should be clearly stated.

2) **Paragraph Structure:** Check that each one of your paragraphs utilizes ample quotes to prove your point.

3) **Misuse of Vocabulary:** Look out for misused words.

4) **Grammatical Errors:** Watch for misplacement or omission of colons, semi-colons, commas, periods, quotation marks, and parentheses. Look for missing words or even sentences.

5) **Spelling Errors:** Make sure you used the proper versions of words like "to," "too," and "two;" "your" and "you're;" "there," "their," and "they're," etc.

6) **Legibility:** If you come across words that you suspect may be hard to read, cross them out neatly and re-write them.

ACTIVITY FOR STEP 4:

Directions: Go back to the essay you wrote in the previous activity and revise it using the above checklist.

LEADING UP TO TEST DAY

You should be planning on taking the SAT two or three times. The vast majority of colleges will either take your best score or will actually sift through your tests to find your best section performances and combine those as if you took them the same testing day (known as a "super score").

Therefore, you have nothing to lose by taking advantage of the fact that the test is offered six times a year. Some colleges will look down on a student who takes the test four or more times, and that is why we suggest taking it just two or three times, total.

EXAM WEEK SCHEDULE

Sunday - Tuesday: This should be the last time you take a full-length practice test if you plan on doing it this late in the game. It's imperative that you understand that this is *just* practice. If your score goes down, that does not mean you are going to fail on test day.

Wednesday - Thursday: For these two days you should start to take it easy. Just review some concepts, look over some old practice questions or tests you did, make sure you review your notes. Look over all the methods, tips and tricks again as well.

Friday: The very last thing to do the day before the test is to put together a final review sheet. This strategy is a fantastic way to review and keep your head in the game the day before the test, as well as the morning of it.

Once you have finished making this review sheet, it should be the *only* thing that you study Friday night and Saturday morning. You want to make sure that you are totally familiar with all the concepts on the sheet. Also, it's a good thing to look at Saturday morning while eating breakfast or waiting in line for the test.

JUST REMEMBER NOT TO TAKE THE SHEET INTO THE TESTING CENTER!!!!
The last thing you need is to be accused of cheating.
This is simply a review sheet, *not* a cheat sheet!

Finally, make sure to plan your trip to the testing center before test day, and maybe even visit it prior to the big day. Friday is often a good day to do this. You don't want to get lost on the morning of the SAT!

WHAT DO I NEED FOR TEST DAY?

It is imperative that you not forget any of the things on this checklist. You want to make sure that you have the following things for test day:

1. A calculator with fresh batteries. (Make sure it's a calculator you are familiar with; it's no fun learning how to use a new device in the middle of the test!) You can find a list of permitted calculators on the College Board website.

2. A watch or stopwatch that doesn't beep! (You never know if the clock will be working or if the proctor will accidentally forget to give you a 2-minute warning.)

3. A few No. 2 pencils with erasers (NOT mechanical pencils)

4. Photo ID card

5. Your admission ticket from College Board

6. A snack.

Good luck and let us know how you did!